

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 112 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (20 分)，請答 O 或 X。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. A horizontal line intersects the graph of an increasing function at most once.
2. If both  $f(x)$  and  $g(x)$  are increasing functions on  $(0, 1)$ , then  $f \circ g$  is also an increasing function on  $(0, 1)$ .
3. If  $f$  is an odd function, then  $\lim_{x \rightarrow 0} f(x) = 0$ .
4. If  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  exists, then  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  both exist.
5. If  $f(x)$  is differential at  $a$ , then  $f$  is continuous at  $a$ .
6. If  $f$  and  $g$  are both differential at  $a$ , then  $\frac{f}{g}$  is also differential at  $a$ .
7.  $|f|' = |f'|$ .
8. The domain of  $f'$  is equal to the domain of  $f$ .
9.  $\frac{d \sin 2x}{dx} = 2 \cdot \frac{d \sin x}{dx}$ .
10. The linear approximation of  $f(x) = \sin x$  at 0 is  $x$ .

(下頁還有試題)

填充題 (32 分)，每題 4 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. If  $f(x) = \frac{1}{\sqrt{x}}$  and  $g(x) = x - 5$ , then the domain of the composite function  $f \circ g$  is \_\_\_\_\_.
2.  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{|x|} \right) =$  \_\_\_\_\_.
3. If  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 5$ , then  $\lim_{x \rightarrow 0} f(x) + \frac{x}{f(x)} =$  \_\_\_\_\_.
4.  $\lim_{t \rightarrow 0} \left( \frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) =$  \_\_\_\_\_.
5. The point-slope equation of the tangent line to the curve  $y = \sqrt{x^2 + 1}$  at  $(0, 1)$  is \_\_\_\_\_.
6. If  $f'(0) = -1$  then  $\lim_{x \rightarrow 0} \frac{f(2x) - f(-x)}{x} =$  \_\_\_\_\_.
7.  $\frac{d^{101}(\sin x)}{dx^{101}} =$  \_\_\_\_\_.
8. Let  $F(x) = f(xf(\sqrt{x}))$ . If  $f(1) = 3$  and  $f'(1) = 5$ , then  $F'(1) =$  \_\_\_\_\_.

(下頁還有試題)

計算問答證明題 (60 分)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (10 points) Prove the statement using the  $\epsilon, \delta$  definition of a limit.

$$\lim_{x \rightarrow 2} \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}}.$$

2. (10 points) Find the values of  $a$  and  $b$  that make  $f$  continuous everywhere

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & \text{if } x < 1. \\ ax^2 + bx + 3, & \text{if } 1 \leq x < 2. \\ 2x + a - b, & \text{if } x \geq 2. \end{cases}$$

3. (10 points) Define  $f(x) = \begin{cases} x^3 \sin \frac{\pi}{x}, & \text{if } x \neq 0. \\ 0, & \text{if } x = 0. \end{cases}$

- For  $x \neq 0$ , find  $f'(x)$ .
- Show that  $f'(0) = 0$
- Show that  $f'(x)$  is continuous for all real numbers  $x$ . (Hint: use **a.** and **b.** )

4. (10 points)

- State the Intermediate Value Theorem .
- Use **a.** to show that there is a root of the equation  $x^4 + x - 3 = 0$  in the interval  $(1, 2)$ .

5. (10 points) Find  $\frac{dy}{dx}$  by implicit differentiation.

$$y \sin x = x \sin(y^2).$$

6. (10 points) Two sides of a triangle have lengths  $12_{cm}$  and  $15_{cm}$ . The angle between them is increasing at a rate of  $2^\circ/min$ . How fast is the length of third side increasing when the angle between the sides of fixed length is  $60^\circ$ ?

(試題結束)