

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (20 分)，請答 O 或 X。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f has an absolute minimal value at c and $f'(c) = 0$, then $f''(c) \geq 0$.
2. If f is continuous and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.
3. There exists a function f such that $f(x) > 0$, $f'(x) > 0$, $f''(x) < 0$ for all x .
4. If f and g are negative increasing functions on an interval I , then fg is increasing on I .
5. If $f(x) = [x]$, then $\int_{-a}^a f(x) dx = 0$ for any real number a .
6. If the definite integral $\int_a^b f(x) dx$ exists, then f is continuous on $[a, b]$.
7. If f has finitely many discontinuities on $[a, b]$, then the definite integral $\int_a^b f(x) dx$ exists.
8. $\int_{-1}^2 5x - x^3 dx$ is equal to the area under the curve $y = 5x - x^3$ from 1 to 2.
9. For all a and b , $|\sin a - \sin b| \leq |a - b|$.
10. $1 \leq \int_{-1}^1 \frac{x+1}{\sqrt{x^2+1}} dx \leq 2$.

(下頁還有試題)

填充題 (40 分)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let M and m be the maximum and minimum value of $f(x) = x\sqrt{x-x^2}$. Then $M + m =$ _____.
2. $\lim_{x \rightarrow \infty} (\sqrt{9x^2 + x} - 3x) =$ _____.
3. The area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is _____.
4. $\int_0^1 (x + \sqrt{1-x^2}) dx =$ _____.
5. Suppose $g(x) = \int_{-x}^x \frac{\sin t + \cos t}{\sqrt{t^2 + 1}} dt$. Then $g'(x) =$ _____.
6. $\int_1^9 \left(\frac{2t^2 + t\sqrt{t} - 1}{\sqrt{t}} \right) dt =$ _____.
7. The area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \frac{\pi}{2}$ is _____.
8. The average value of $f(x) = \frac{1}{(1 + \sqrt{x})^2}$ on $[0, 4]$ is _____.

(下頁還有試題)

計算問答證明題 (60 分)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (10 points) State the Fundamental Theorem of Calculus (Part I and Part II).
2. (10 points) Use the definition to prove that $\lim_{x \rightarrow \infty} \frac{x}{x+1} = 1$.
3. (10 points) Let $f(x) = \frac{x^2}{x-1}$.
 - a. Find the intervals on which f is increasing or decreasing.
 - b. Find the intervals of concavity and the inflection points.
 - c. Find the vertical, horizontal, and slant asymptotes of $y = f(x)$.
 - d. Sketch the curve $y = f(x)$.
4. (10 points) A piece of wire $10m$ long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) A maximum? (b) A minimum?
5. (10 points) Represent the volume of a right circular cone with height h and base radius r as a definite integral. Evaluate the definite integral.
6. (10 points) Let S be the solid obtained by rotating about $x = -1$ the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

(試題結束)