

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。

是非題 (20 分)，請答 O 或 X。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f is one-to-one, with range R , then $f(f^{-1}(0)) = 0$.
2. If $f : R \rightarrow R$ is increasing and differentiable everywhere, then $(f^{-1})'(x) > 0$ for all x .
3. $\ln x^6 = 6 \ln x$ for all x .
4. $\int_{-2}^{-16} \frac{dx}{x} = 3 \ln 2$.
5. $\lim_{x \rightarrow 0} \frac{\ln(\frac{1}{1+x})}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{(\frac{-1}{1+x})}{\sin x} = \lim_{x \rightarrow 0} \frac{\frac{1}{(1+x)^2}}{\cos x} = 1$.
6. $\int_0^1 \frac{1}{x^2 - 1} dx$ is convergent.
7. $\int_{-\infty}^{\infty} \frac{1}{x^3 + 1} dx$ is convergent.
8. Every elementary function has an elementary antiderivative.
9. If f is continuous on $[-1, 1]$, then $\int_{-1}^1 f(x) dx = \int_{-1}^1 f(-x) dx$.
10. $\cos(\sec^{-1} x) = \frac{1}{x}$ for all $x \neq 0$.

(下頁還有試題)

填充題 (40 分)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. $\lim_{x \rightarrow \infty} \ln(2+x) - \ln(1+x) = \underline{\hspace{2cm}}$.

2. If $f(x) = 3 + x + e^x$, then $(f^{-1})'(4) = \underline{\hspace{2cm}}$.

3. $\int_0^1 \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \underline{\hspace{2cm}}$.

4. $\lim_{x \rightarrow 0^+} \frac{x^x - 1}{\ln x + x - 1} = \underline{\hspace{2cm}}$.

5. The area of the region bounded by the curves $y = x^2 \ln x$ and $y = 4 \ln x$ is
 $\underline{\hspace{2cm}}$.

6. The average value of $f(x) = \frac{\sqrt{x^2+1}}{x}$ on $[1, 7]$ is $\underline{\hspace{2cm}}$.

7. $\int_0^9 \frac{1}{\sqrt[3]{x-1}} dx = \underline{\hspace{2cm}}$.

8. $\frac{d}{dx}(\tan^{-1} x)|_{x=0} = \underline{\hspace{2cm}}$.

(下頁還有試題)

計算問答證明題 (60 分)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (10 points) The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample.
 - a. Find the mass that remains after t years.
 - b. How much of the sample remains after 100 years ?
 - c. After how long will only 1 mg remain ?

2. (10 points) Let

$$f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0. \\ 0, & \text{if } x = 0. \end{cases}$$

Use the definition of derivative to $f'(0)$.

3. (10 points) Evaluate the integral.

$$\int \frac{x \ln x}{\sqrt{x^2 - 1}} dx$$

4. (10 points) Determine the improper integral $\int_{-a}^a \frac{dx}{\sqrt{a^2 - x^2}}$ is convergent or divergent. Evaluate it if it is convergent.

5. (10 points) Find the length of the curve

$$\int_1^x \sqrt{t^3 - 1} dt, \quad 1 \leq x \leq 4.$$

6. (10 points) If the infinite curve $y = e^{-x}$, $x \geq 0$, is rotated about the x-axis, find the area of resulting surface.

(試題結束)