微積分聯合教學　段考四　**Mar 26, 2013**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The series $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} n$ is convergent.

2. If $\displaystyle\lim_{n\to\infty} a_n = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is covergent.

3. There exists a power series $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n$ whose interval of convergence is $[0,\infty)$.

4. Define $a_{n+1} = 4 - a_n$ for $n \geq 1$. Then $\{a_n\}$ must be convergent.

5. Let $\displaystyle f(x) = \sum_{n=0}^{\infty} (-1)^n \left[\frac{x^{2n}}{(2n)!} + \frac{x^{2n+1}}{(2n+1)!}\right]$, then $f\left(\frac{\pi}{4}\right) = \sqrt{2}$.

6. If $a_n > 0$, $n \geq 1$, $\displaystyle\lim_{n\to\infty} a_n = L$, $-\infty < L < \infty$ and $L \neq 0$, then $\displaystyle\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ is divergent.

7. If $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, then $\displaystyle\sum_{n=1}^{\infty} {a_n}^2$ is also convergent.

8. The equality $\displaystyle e^{x^2+2x} = \sum_{n=0}^{\infty} \frac{1}{n!e}(x+1)^{2n}$ holds for all $x \in R$.

9. The two polar curves $r = -1$ and $r = \cos^2\theta$ have no intersection points.

10. If $\displaystyle\lim_{n\to\infty} a_n = L$, then $\displaystyle\lim_{n\to\infty} a_{2n+1} = L$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find a parametric equation for the path of a particle that moves along the circle $x^2 + (y-1)^2 = 4$ three times around counterclockwise, starting at $(2, 1)$.

Answer：___________.

2. Find the slope of the tangent to the cycloid $x = 2(\theta - \sin\theta)$, $y = 2(1 - \cos\theta)$ at the point where $\theta = \dfrac{\pi}{4}$.

Answer：___________.

3. Find the length of the curve $r^2 = 1$, $0 \leq \theta \leq \frac{\pi}{2}$.

Answer：___________.

4. Consider the following two curves

$$C_1 : r = 1 + \sin\theta, \quad 0 \leq \theta \leq 2\pi.$$

$$C_2 : r = 1 - \cos\theta, \quad 0 \leq \theta \leq 2\pi.$$

If the length of C_1 is 8, find the length of C_2.

Answer：___________.

5. If $x = t^2 + 1$, $y = t^2 + t$, then find $\dfrac{d^2y}{dx^2}$.

Answer：___________.

6. If 2 is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} c_n (x-a)^n$, and $\displaystyle\lim_{n\to\infty} \left| \frac{C_{n+1}}{C_n} \right|$ exists, then find the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} n \cdot {c_n}^2 (x-a)^n$.

Answer：___________.

7. Find the Taylor series of $f(x) = \dfrac{1}{x^2}$ centered at -3.

Answer：___________.

8. Find the surface area generated by rotating the curve $x = 3t^2$, $y = 2t^3$, $0 \leq t \leq 5$ about the y-axis.

Answer：___________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points)

a. Find a power series representation for $g(x) = \dfrac{1}{5-x}$ and determine the radius of convergence.

b. Find a power series representation for $f(x) = \ln(5-x)$ by using **a**. What is the radius of convergence of the power series for $\ln(5-x)$.

2. (*10* points) Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{n+1}}{3^n \ln(n+1)}$$

3. (*10* points) For what values of r is the sequence $\{nr^n\}$ convergent?

4. (*10* points) Determine whether the series is convergent or divergent.

a. $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$

b. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$

5. (*10* points) Determine whether the series is convergent or divergent.

a. $\sum_{n=1}^{\infty} \dfrac{\ln n}{n^3}$

b. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

6. (*10* points)

a. Sketch the curve with the given polar equation.

$$r = 3 - 2\sin\theta$$

b. Find the area that it encloses.

(試題結束)