

考試時間 120 分鐘，題目卷為三張紙，共五頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If  $f$  is a polynomial function of degree 2, then Simpson's rule gives the exact value of  $\int_a^b f(x)dx$ .
2. If  $\lim_{N \rightarrow \infty} \int_{-N}^N f(x)dx$  exists, then  $\int_{-\infty}^{\infty} f(x)dx$  converges.
3. If the region bounded by  $y = x^2$ , the  $y$ -axis,  $y = 1$  in the first quadrant is revolved about the  $x$ -axis, the volume generated is  $V = \pi \int_0^1 (1 - x^2)^2 dx$ .
4. The equation of the level curve of  $f(x, y) = xy$  that contains the point  $\left(\frac{1}{4}, 4\right)$  is  $xy = 1$ .
5. Let  $C$  be the curve of intersection of the surface  $z = \sqrt{x^2 - y^2}$  with the plane  $x = 3$ . Then the slope of the tangent line to  $C$  at the point  $(3, 1, \sqrt{8})$  is  $\frac{3}{\sqrt{8}}$ .
6. If  $f_x(a, b) = 0$  and  $f_y(a, b) = 0$ , then  $f$  may not have a relative extremum at the point  $(a, b)$ .
7. Given  $n$  data points  $(x_1, y_1), \dots, (x_n, y_n)$ , the least-squares line for the data is given by the equation  $y = mx + b$ , where  $m$  and  $b$  satisfy  $(x_1^2 + \dots + x_n^2)m + (x_1 + \dots + x_n)b = x_1y_1 + \dots + x_ny_n$ , and  $(x_1 + \dots + x_n)m + nb = y_1 + \dots + y_n$ .

(下頁還有試題)

8. If  $(a, b)$  gives rise to a (constrained) relative extremum of  $f$  subject to the constraint  $g(x, y) = 0$ , then  $f_x(a, b) = 0$  and  $f_y(a, b) = 0$ , simultaneously.
9. Suppose  $f$  and  $g$  have continuous first partial derivatives in some region  $D$  in the plane. If  $f$  has an extremum at a point  $(a, b)$  subject to the constraint  $g(x, y) = c$ , then there exists a constant  $\lambda$  such that  $f_x(a, b) = -\lambda g_x(a, b)$ ,  $f_y(a, b) = -\lambda g_y(a, b)$ , and  $g(a, b) = c$ .
10. The total differential of  $\omega = f(x, y, z) = y\sqrt{z} + \ln(x^2 + y^2)$  is  

$$d\omega = \frac{2x}{x^2 + y^2} dx + \left( \sqrt{z} + \frac{2y}{x^2 + y^2} \right) dy + \frac{y}{2\sqrt{z}} dz.$$

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. A company determines that its marginal cost, in dollars, for producing  $x$  units of a product is given by  $C'(x) = 3600x^{-1.8}$ , where  $x \geq 1$ . Suppose that it were possible for the company to make infinitely many units of this product. What would the total cost be?      Answer : \_\_\_\_\_.
2. Find the volume of the solid of revolution formed by rotating the region under the graph of  $y = \frac{1}{x}$  over the interval  $[1, \infty)$  about the  $x$ -axis.  
 Answer : \_\_\_\_\_.
3. The demand equations of two commodities are  $x = f(p, q) = \frac{2q}{1 + \sqrt{p}}$  and  $y = g(p, q) = \frac{3p}{1 + q^2}$ , respectively. Determine whether these two products are substitute, complementary, or neither.      Answer : \_\_\_\_\_.
4. Find the critical point(s) of the function  $f(x, y) = xy + \ln x + 50y^2$ . Then use the second derivative test to classify the nature of each point.  
 Answer : \_\_\_\_\_.
5. Find the area of the region bounded by the  $x$ -axis and the graph of the function  $f(x) = \frac{xe^{-x^2}}{1 + e^{-x^2}}$ .      Answer : \_\_\_\_\_.

(下頁還有試題)

6. If  $f(x, y) = x^2y^3 + x^4y + (\ln x)e^y$ , evaluate  $f_x(1, 1)$  and  $f_{yx}(1, 1)$ .

Answer : \_\_\_\_\_.

7. The production of an East Asian country is given by the function  $f(x, y) = 40x^{0.75}y^{0.25}$  when  $x$  units of labor and  $y$  units of capital are utilized. Find the approximate change in output if the amount expended on labor is decreased from 256 to 254 units and the amount expended on capital is increased from 16 to 18 units. Answer : \_\_\_\_\_.

8. The management of UNICO Department Store decides to enclose an  $600 \text{ ft}^2$  area outside their building to display potted plants. The enclosed area will be a rectangle, one side of which is provided by the external walls of the store. Two sides of the enclosure will be made of pine board, and the fourth side will be made of galvanized steel fencing material. If the pine board fencing costs  $\$9/\text{running foot}$  and the steel fencing costs  $\$3/\text{running foot}$ , determine the dimensions of the enclosure that will cost the least to erect.

Answer : \_\_\_\_\_.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) A firm produces two kinds of golf ball, one that sells for  $\$3$  and one priced at  $\$2$ . The total revenue, in thousands of dollars, from the sale of  $x$  thousand balls at  $\$3$  each and  $y$  thousand at  $\$2$  each is given by  $R(x, y) = 3x + 2y$ . The company determines that the total cost, in thousands of dollars, of producing  $x$  thousand of the  $\$3$  ball and  $y$  thousand of the  $\$2$  ball is given by

$$C(x, y) = 2x^2 - 2xy + y^2 - 9x + 6y + 7.$$

How many balls of each type must be produced and sold in order to maximize profit?

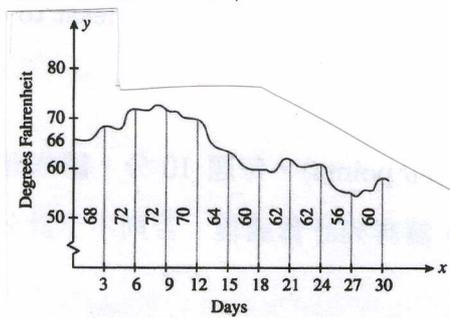
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2. (10 points) A consulting firm for a manufacturing company arrived at the following Cobb Douglas production function for a particular product:  $f(x, y) = 50x^{0.8}y^{0.2}$ . In this equation,  $x$  is the number of units of labor and  $y$  is the number of units of capital required to produce  $f(x, y)$  units of the product. Each unit of labor costs \$40 and each unit of capital costs \$80.

(a) If \$400,000 is budgeted for production of the product, use the method of the Lagrange multipliers to determine how that amount should be allocated to maximize production, and find the maximum production.

(b) Find the marginal productivity of money in this case, and estimate the increase in production if an additional 50,000 is budgeted for the production of the product.

3. (10 points) The graph depicted in the following figure shows the daily mean temperatures recorded during one September in Cameron Highlands. Using Simpson's rule with  $n = 10$ , estimate the average temperature during that month.



4. (10 points) Becky Wilkinson wants to establish a trust fund that will provide her children and heirs with a perpetual annuity in the amount of  $P(t) = 20 + t$  thousand dollars/year beginning next year. If the trust fund will earn an interest rate of 10%/year compounded continuously, find the amount that she must place in the trust fund now.

(Hint: (1) The present value of a perpetual annuity is given by  $PV \approx \int_0^{\infty} P(t)e^{-rt} dt$ , where  $r$  is the interest rate compounded continuously. (2)  $\lim_{b \rightarrow \infty} \frac{b}{e^{rb}} = 0$ .)

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5. (10 points) The productivity of a certain country is given approximately by the function

$$f(x, y) = 10x^{0.25}y^{0.75}$$

with the utilization of  $x$  units of labor and  $y$  units of capital.

(a) Find  $f_x(x, y)$  and  $f_y(x, y)$ .

(b) If the country is now using 600 units of labor and 100 units of capital, find the marginal productivity of labor and the marginal productivity of capital.

(c) For the greatest increase in the country's productivity under the condition as that in (b), should the government encourage increased use of labor or increased use of capital?

6. (10 points) The flow of blood through an arteriole in cubic centimeters per second is given by

$$V = \frac{\pi pr^4}{8kl}$$

where  $l$  is the length (in cm) of the arteriole,  $r$  is its radius (in cm),  $p$  is the difference in pressure between the two ends of the arteriole (in dyne/cm<sup>2</sup>), and  $k$  is the viscosity of blood (in dyne-sec/cm<sup>2</sup>). Find the approximate percentage change in the flow of blood if an error of 2% is made in measuring the length of the arteriole and an error of 1% is made in measuring its radius. Assume that  $p$  and  $k$  are constant.

(試題結束)