微積分聯合教學　段考五　**May 07, 2013**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. For any vectors u and v in V_3, $|u \times v| = |v \times u|$.

2. In R^3, the graph $z = x^2$ is a paraboloid.

3. Suppose f is twice continuously differentiable. At an inflection point of the curve $y = f(x)$, the curvature is 0.

4. The curve with vector equation $\mathbf{r}(t) = t^3\mathbf{i} + 2t^3\mathbf{j} + 3t^3\mathbf{k}$ is a line.

5. If $f(x) = \ln y$, then $\nabla f(x, y) = \dfrac{1}{y}$.

6. The distance from the point $P(1, 1, 1)$ to the line through $Q(0, 6, 8)$ and $R(-1, 4, 7)$ is $\dfrac{\sqrt{271}}{3}$.

7. There is no function f where partial derivatives are $f_x(x, y) = x + 4y$ and $f_y(x, y) = 3x - y$.

8. If $|\mathbf{r}(t)| = 1$ for all t, then $|\mathbf{r}'(t)|$ is a constant.

9. If f has a local minimum at (a, b), then $\nabla f(a, b) = 0$.

10. Let C be the circle traced by the vector function $\mathbf{r}(t) = \langle \cos 3t, \sin 3t \rangle$, then the circumference of C is $\displaystyle\int_0^{2\pi} |\mathbf{r}'(t)|\, dt$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Match the parametric equations with the graphs (labeled I–VI). One point for each correct match.

1.1 $x = t,\ y = 1/(1+t^2),\ z = t^2$.

1.2 $x = \cos 4t,\ y = t,\ z = \sin 4t$.

1.3 $x = t,\ y = t^2,\ z = e^{-t}$.

1.4 $x = \cos t,\ y = \sin t,\ z = \sin 5t$.

1.5 $x = e^{-t}\cos 10t,\ y = e^{-t}\sin 10t,\ z = e^{-t}$.

Answer：____________.

2. Find the symmetric equations of the normal line to $x+y+z = e^{xyz}$ at $(0,0,1)$.

Answer：____________.

3. Find the unit vectors that are parallel to the tangent line to the curve $y = 2\sin x$ at the point $(\pi/3, \sqrt{3})$. Answer：____________.

4. Find the domain of $f(x,y) = \dfrac{\sqrt{y-x^2}}{1-x^2}$. Answer：____________.

5. At what point on the curve $x = t^3,\ y = 3t,\ z = t^4$ is the normal plane parallel to the plane $6x + 6y - 8z = 1$. Answer：____________.

6. The plane that passes through the point $(1,2,-1)$ and contains the line of intersection of the planes $x + y - z = 2$ and $x - y + 3z = 1$.

Answer：____________.

7. Find the linearization $L(x,y)$ of the function $f(x,y) = e^{-xy}\cos y$ at $(\pi, 0)$.

Answer：____________.

8. Find the directional derivative of $f(x,y) = xe^y + \cos xy$ at $(2,0)$ in the direction of $\langle 3, -4\rangle$. Answer：____________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the local maximum and minimum values and saddle point(s) of the function $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

2. (*10* points)

a. Show that the lines L_1 and L_2 with parametric equations.

$$x = 1+t,\ y = -2+3t,\ z = 4-t,\ t \in R$$

$$x = 2s,\ y = 3+s,\ z = -3+4s,\ s \in R$$

are skew lines.

b. Find the distance between them.

3. (*10* points) Find

$$\lim_{(x,y)\to(\frac{3}{2},0)} \frac{2xy-3y}{(2x-3)^2+y^2}$$

if it exists, or show that the limit does not exist.

4. (*10* points) **a.** Let $f : R \to R$ and $F : R^2 \to R$ be differentiable and satisfy $F(x, f(x)) = 0$ and $F_y \neq 0$. Prove that $f'(x) = -\dfrac{F_x}{F_y}$ where $y = f(x)$.

b. Find $\dfrac{dy}{dx}$ by implicit function theorem if $x^3 + y^3 = 6xy$.

5. (*10* points) $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & ,\ (x,y) \neq (0,0) \\ 0 & ,\ (x,y) = (0,0) \end{cases}$

a. Show that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not differentiable at $(0,0)$.

b. Explain why f_x and f_y are not continuous at $(0,0)$.

6. (*10* points) $\mathbf{r} = \langle t, 3\cos t, 3\sin t \rangle$

a. Find the unit tangent and unit normal vector $\mathbf{T}(t)$, $\mathbf{N}(t)$, and $\mathbf{B}(t)$.

b. Find the curvature.

(試題結束)