管院微積分聯合教學　段考五　**May 07, 2013**

考試時間 120 分鐘，題目卷為三張紙，共五頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Let R be the region bounded by the lines $x = 0$, $y = 1$, and $y = x$. Then

$$\iint_R xe^y dA = \int_0^1 \int_x^1 xe^y dydx = \int_0^1 \int_y^1 xe^y dxdy.$$

2. The following equality

$$\int_0^2 \int_{y/2}^{y} \sqrt{x}e^{y^2} dxdy + \int_2^4 \int_{y/2}^{2} \sqrt{x}e^{y^2} dxdy = \int_0^2 \int_x^{2x} \sqrt{x}e^{y^2} dydx$$

holds.

3. If $y = f(x)$ is a solution of a first-order differential equation, then $y = f(x) + C$ (C, a constant) is also a solution.

4. The function $f(x, y) = xy$ is a joint probability density function on

$$D = \{(x, y)|0 \le x \le 1,\ 0 \le y \le 1\}.$$

5. Given that $f(x) = \frac{1}{9}xe^{-x/3}$ is the probability density function of a random variable X on $[0, \infty)$, $E(X) = 6$, and $\mathrm{Var}(X) = 18$. Then

$$\int_0^\infty \frac{1}{9}x^3 e^{-x/3} dx = 54.$$

(下頁還有試題)

6. Suppose X is a normal random variable with $\mu = 100$ and $\sigma = 20$. Then

$$P(100 < X < 120) = \frac{1}{2} - P(Z < 1)$$

where Z is the standard normal random variable.

7. Suppose f is a function over the plane region R. Then the average value of f over R can be thought of as the constant height of the cylinder with base R and volume that is exactly equal to the volume of the solid under the graph of $z = f(x, y)$.

8. Use Euler's method with $n = 5$ to obtain approximations to the solution of the initial value problem

$$y' = 2x + y, \quad y(0) = 1$$

when $x = 1$. It is known that $y(0.8) \approx \frac{1638}{625}$. Then

$$y(1) \approx \frac{1638}{625} + 0.2\left(1.6 + \frac{1638}{625}\right).$$

9. Suppose glucose is infused into bloodstream at a constant rate of C g/min, and at the time, the glucose is converted and removed from the bloodstream at a rate proportional to the amount of glucose present. Then the amount of glucose $A(t)$ present in the bloodstream at any time t is governed by the differential equation

$$\frac{dA}{dt} = C - kA$$

where k is a constant.

10. The population density of a coastal town is given by the function

$$f(x, y) = \frac{10000e^{|y|}}{1 + 0.5x}, \; 0 \le x \le 4, -10 \le y \le 10$$

where x and y are measured in miles. Then the population of this town is

$$2\int_{-10}^{0}\int_{0}^{4} \frac{10000e^{-y}}{1 + 0.5x} dxdy.$$

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填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the solution of the initial value problem $y' = \dfrac{xy}{x^2+1}; y(0) = 1$.

Answer：___________.

2. Let $R = [0,1] \times [0,1]$, then from geometrical point of view, the double integral $\int\int_R (4-2y)dA$ represents ___________. Answer：___________.

3. Use Euler's method with $n = 3$ to obtain an approximation of the solution of the initial value problem $y' = x - 2y, \quad y(0) = 1$, when $x = 1$. Answer：___________.

4. The life span of a certain plant spices (in days) is described by an exponential distributed random variable X with expected value of 100 (days). What is the probability of a plant selected at random that will survive at least 100 days? Answer：___________.

5. Consider the model for restricted growth described by the differential equation

$$\frac{dQ}{dt} = 0.0038Q(2250 - Q)$$

where $Q(t)$ denotes the size of a certain population at time t. If $Q(0) = 125$, find the size at which the rate of growth of Q is greatest. Answer：___________.

6. An expert says that the amount of rainfall (in inches) on a tropical island in the month of August is a continuous random variable with probability density function $f(x) = \dfrac{1}{16}x(6-x)$, $0 \le x \le 6$. Do you agree with his statement? Why? Answer：___________.

7. Find the volume of the solid bounded above by the surface $z = f(x,y) = \dfrac{y}{x^3+2}$ and below by the region which is bounded by the lines $x = 1$, $y = 0$, and $y = x$. Answer：___________.

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8. The population of certain spices grows at a rate proportional to the square root of its size. Let $N(t)$ denote the size of the population at any time t. If the initial populaiton is N_0, formulate the problem in terms of a differential equation with a side condition. Do not solve it. Answer：___________.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) The scores on an economics examination are normally distributed with a mean of 72 and a standard deviation of 16. If the instructor assigns a grade of A to 10% of the class, what is the lowest score a student may have and still obtain an A? (Hint: $P(Z < -1.28) \approx 0.10$, where Z is the standard normal random variable.)

2. (*10* points) A tank initially contains 50 gal of brine, in which 10 lb of salt is dissolved. Brine containing 2 lb of dissolved salt per gallon flows into the tank at the rate of 2 gal/min, and the well-stirred mixture flows out the tank at the same rate.

(a) How much salt is present in the tank at any time t?

(b) How much salt is present in the tank at the end of 10 min?

(c) How much salt is present in the long run?

3. (*10* points) An amount of money deposited in a saving account grows at a rate proportional to the amount present. Suppose $10000 is deposited in an account earning interest at the rate of 10% year compounded continuously.

(a) What is the accumulated amount after 5 years?

(b) How long does it take for the original deposit to double in value?

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4. (*10* points) Let $f(x, y) = ke^{-x-3y}$ be the joint probability density function for the random variables X and Y on $D = \{(x, y)|x \geq 0 \text{ and } y \geq 0\}$.

(a) Find the constant k. **(b)** Find $P(2X < Y)$.

5. (*10* points) Evaluate the following integrals.

(a) $\displaystyle\int_0^{\ln 5} \int_{e^x}^{5} \frac{1}{\ln y} dy dx$

(b) $\displaystyle\iint_R \sqrt{3 - x^2 - y^2} dA$, where $R = \{(x, y)|x^2 + y^2 \leq 3\}$.

6. (*10* points) Consider a differential equation $\dfrac{dy}{dt} = \dfrac{k}{v}(C - y)$, $y(0) = y_0$, where k, v, C and y_0 are positive constants with $C - y > 0$. Find the limit of y as t tends to infinity and interpret your result.

(試題結束)