微積分聯合教學　段考六　**Jun 18, 2013**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is covergent.

2. There is no function f where partial derivatives are $f_x(x,y) = x + 4y$ and $f_y(x,y) = 3x - y$.

3. There is vector field $\mathbf{F}$ such that curl $\mathbf{F} = x\mathbf{i} - y\mathbf{j} + x\mathbf{k}$.

4. If C is any closed path on R^2 then $\int_C e^x \sin y dx + e^x \cos y dy = 0$.

5. $\int_0^1 \int_0^x \sqrt{x+y^2} dydx = \int_0^x \int_0^1 \sqrt{x+y^2} dxdy$.

6. $\int_1^4 \int_0^1 (x^2 + \sqrt{y}) \sin(x^2y^2) dxdy \leq 9$.

7. The integral $\int_0^{2\pi} \int_0^2 \int_r^2 dzdrd\theta$ represents the volume enclosed by the cone $z = \sqrt{x^2+y^2}$ and the plane $z = 2$.

8. If f has continuous partial derivative on R^3 and C is any circle, then $\int_C \nabla f \cdot d\mathbf{r} = 0$.

9. If $\mathbf{F}$ and $\mathbf{G}$ are vector fields, then curl $(\mathbf{F}\cdot\mathbf{G}) =$ curl $\mathbf{F}\cdot$ curl $\mathbf{G}$.

10. $\int_{-1}^1 \int_0^1 e^{x^2+y^2} \sin y dxdy = 0$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Determine whether $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ is convergent or divergent.

Answer ： ___________.

2. Find the volume of the solid lying under the surface $z = xy$ and above the triangle with vertices $(1, 1)$, $(4, 1)$, and $(1, 2)$.

Answer ： ___________.

3. Match the functions $f(x, y) = (x + y)^2$ with the plots of their gradient vector fields labeled I-IV.

Answer ： ___________.

4. The density at any point on a semicircular lamina is proportional to the distance from the center of the circle. Find the center of mass of the lamina.

Answer ： ___________.

5. Find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$. Answer ： ___________.

6. Evaluate the iterated integral $\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \sin(x^2 + y^2)\,dy\,dx$ by converting to polar coordinates. Answer ： ___________.

(下頁還有試題)

7. Convert the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dzdydx$$

to an equivalent integral in spherical coordinates. (**Do not evaluate the integral.**) Answer ： ___________.

8. Evaluate the line integral $\int_C (2x+9z)ds$, where $C: x=t, y=t^2, z=t^3, 0 \le t \le 1$. Answer ： ___________.

計算問答證明題 **Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) At what point on the curve $x=t^3$, $y=3t$, $z=t^4$ is the normal plane parallel to the plane $6x+6y-8z=1$.

2. (*10* points) Evaluate the integral by making an appropriate change of variables.

$$\int\int_R e^{x-y} \, dA,$$

where R is given by the inequality $|x|+|y| \le 1$.

3. (*10* points)

a. Show that $\mathbf{F}(x,y,z) = y^2z^3\mathbf{i} + 2xyz^3\mathbf{j} + 3xy^2z^2\mathbf{k}$ is a conservative vector field.

b. Find a function f such that $\mathbf{F} = \nabla f$.

4. (*10* points) Evaluate the iterated integral $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dxdy$.

5. (*10* points) Use the Green's Theorem to evaluate the line integral

$$\oint_C (y+e^{\sqrt{x}})dx + (2x+\cos y^2)dy,$$

where C is the boundary of the region enclosed by the parabolas $y=x^2$ and $x=y^2$.

(試題結束)