

考試時間 120 分鐘，題目卷為二張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\{a_n\}$ and $\{a_n + b_n\}$ converge, then $\{b_n\}$ also converges.
2. If $a_n \leq b_n \leq 0$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges.
3. If $0 \leq na_n \leq \frac{5}{n}$, then $\sum_{n=1}^{\infty} a_n$ converges.
4. $\sum_{n=1}^{\infty} (\sqrt{10} - 2)^n = \frac{\sqrt{10} - 2}{3 - \sqrt{10}}$.
5. If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.
6. If $\sum_{n=0}^{\infty} a_n(x - 4)^n$ converges at $x = -2$, then it also converges at $x = 9$.
7. $\frac{d^6}{dx^6}(\cos x) = \cos x$.
8. $\int_{-\frac{\pi}{4}}^0 \sqrt[3]{x} \sin 4x dx \geq 0$.
9. $\lim_{x \rightarrow 0} \frac{x^2}{\cos x} = \lim_{x \rightarrow 0} \frac{2x}{-\sin x} = -2$.
10. If $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$ for $|x - a| < R$, then
 $f'(x) = \sum_{n=0}^{\infty} \frac{f^{(n+1)}(a)}{n!} (x - a)^n$ for $|x - a| < R$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the volume of the solid bounded above by the surface $z = f(x, y) = \frac{y}{x^3 + 2}$ and below by the region which is bounded by the lines $x = 1$, $y = 0$, and $y = x$.

Answer : _____.

2. Find the critical point(s) of the function $f(x, y) = xy + \ln x + 50y^2$. Then use the second derivative test to classify the nature of each point.

Answer : _____.

3. If $r = \sum_{n=1}^{\infty} \left(\frac{3}{8}\right)^n$, then $\sum_{n=0}^{\infty} r^n =$ _____. Answer : _____.

4. Assume that $\frac{1}{2-x} = \sum_{n=0}^{\infty} (x-1)^n$, $0 < x < 2$. Find the power series for $\ln \frac{1}{2-x}$ in the interval $(0, 2)$. Answer : _____.

5. When we use Newton's method to approximate the zero of $f(x) = x^3 - x - 1$ in the interval $(1, 2)$ with $x_0 = 1.3$, the value of x_1 is _____.

Answer : _____.

6. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2n(x-3)^n}{(n+1)!}$.

Answer : _____.

7. Find an equation of the tangent line to the graph of the function

$$f(x) = e^{\sec x}$$

at the point $\left(\frac{\pi}{4}, e^{\sqrt{2}}\right)$. Answer : _____.

8. Evaluate the limit using *L'Hôpital's* rule, if appropriate $\lim_{x \rightarrow 0} \frac{\ln(x^2 + 1)}{\cos x - 1}$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Becky Wilkinson wants to establish a trust fund that will provide her children and heirs with a perpetual annuity in the amount of $P(t) = 20 + t$ thousand dollars/year beginning next year. If the trust fund will earn an interest rate of 10%/year compounded continuously, find the amount that she must place in the trust fund now.

(Hint:(1)The present value of a perpetual annuity is given by $PV \approx \int_0^{\infty} P(t)e^{-rt} dt$, where r is the interest rate compounded continuously. (2) $\lim_{b \rightarrow \infty} \frac{b}{e^{rb}} = 0$.)

2. (10 points) Determine whether the series is convergent or divergent.

(a) $\sum_{n=9}^{\infty} \frac{1}{n(\ln n)^3}$. (b) $\sum_{n=1}^{\infty} \frac{e^{2/n}}{n^2}$.

3. (10 points) Use the eighth-degree Taylor polynomial of $f(x) = \frac{1}{(1+x^2)}$ at $x = 0$ and the relationship

$$\pi = 4 \int_0^1 \frac{dx}{1+x^2}$$

to obtain an approximation of π .

(下頁還有試題)

4. (10 points) The revenue of McMenemy's Fish Shanty, located at a popular summer resort, is approximately

$$R(t) = 2 \left(5 - 4 \cos \frac{\pi}{6} t \right), \quad (0 \leq t \leq 12)$$

during the t^{th} week ($t = 0$ corresponds to the first week of June), where R is measured in thousands of dollars. What is the total revenue realized by the restaurant over the 12-wk period starting June 1?

5. (10 points) Jerry wants to establish a scholarship fund at his alma mater, NCU (No Kidding). If a scholarship in the amount of \$10,000 is to be awarded on an annual basis beginning next year, find the amount of the endowment he is required to make now. Assume that this fund will earn interest at a rate of 1% per year compounded continuously.

6. (10 points) The revenue of Long John Silver's restaurant located at Nowhere, Taiwan is approximately

$$R(t) = 3 \left(5 - 2 \cos \frac{\pi t}{3} \right), \quad (0 \leq t \leq 6)$$

during the t^{th} week ($t = 1$ corresponds to the first week of June), where R is measured in thousands of dollars.

- (a) When does the weekly revenue have a maximum?
(b) When is the weekly revenue increasing most rapidly?

(試題結束)