

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f(x) = x^2$ and $g(x) = \sqrt{x}$, then $(f \circ g)(x) = (\sqrt{x})^2 = x$. Thus, the domain of $f \circ g$ is $[0, \infty)$.
2. If $f(x) = \sqrt{x}$, then $f(x)$ is differentiable.
3. Let $f(x) = \lfloor x \rfloor$ and $g(x) = x - \lfloor x \rfloor$, then $f(g(x)) = g(f(x))$.
4. If f and g both are odd functions, then $f(g(x))$ is an even function.
5. $\frac{d}{dx}|x|^4 = 4|x|^3$.
6. If functions f and g are continuous at $x = 0$, then the composite $f \circ g$ is also continuous at $x = 0$.
7. The derivative of
$$h(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0. \\ 0, & \text{if } x = 0. \end{cases}$$
is continuous at $x = 0$.
8. Suppose that $f(x) = x^2$ and $g(x) = |x|$. Then the composites $f \circ g$ and $g \circ f$ are both differentiable at $x = 0$.

(下頁還有試題)

9. The curve $f(x) = x \sin(1/x)$ has a horizontal asymptote.
10. Given $\sec 0 = 1$ and $\sec \pi = -1$, by the *intermediate value theorem* there exists a number $c \in (0, \pi)$ such that $\sec c = 0$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $f(x) = x - 3$, $g(x) = \sqrt{x}$, $h(x) = x^5$, $j(x) = 2x$. Express $u = \sqrt{x^5 - 3}$ as a composite function involving one or more of f , g , h and j .

Answer : _____.

2. Find $\frac{d^{25}}{dx^{25}}(\cos(2x))$.

Answer : _____.

3. If $\lim_{x \rightarrow 0^+} f(x) = A$, $\lim_{x \rightarrow 0^-} f(x) = B$. Find $\lim_{x \rightarrow 0^-} f(x^4 - x^2)$.

Answer : _____.

4. Find the slope to the curve

$$f(x) = \begin{cases} \frac{1 - \cos x}{x}, & \text{if } x \neq 0. \\ 0, & \text{if } x = 0. \end{cases}$$

at $x = 0$. Answer : _____.

5. Find an equation for the line perpendicular to the tangent to the curve $y = x^3 - 4x + 1$ at the point $(2, 1)$.

Answer : _____.

6. Find $\lim_{\theta \rightarrow \pi/6} \frac{\sin \theta - \frac{1}{2}}{\theta - \frac{\pi}{6}}$.

Answer : _____.

7. Assume that $f(0) = 9$, $f'(0) = -2$, $f(1) = -3$, $f'(1) = 1/5$. Find the first derivative of the combinations $f(1 - 5 \tan x)$ at $x = 0$.

Answer : _____.

8. Find $f'(x)$ if it is known that $\frac{d}{dx}[f(2x)] = x^2$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points)

a. (5 points) If $f'(0) = -2$, find $\lim_{h \rightarrow 0} \frac{f(3h) - f(-2h)}{h}$.

b. (5 points) Find $\lim_{x \rightarrow 0} \left[\left(\cos \frac{1}{x} \right) (\sin x) \right]$.

2. (10 points) Find $\frac{d^2y}{dx^2}$ by **implicit differentiation** if $2x^3 - 3y^2 = 8$.

3. (10 points) Coffee is drain(排出) from a conical filter(濾器) into a cylindrical coffee pot at the rate $160 \text{ cm}^3/\text{min}$. (The volume of the circular cone with base radius r and height h is $V = \frac{1}{3}\pi hr^2$.)

a. How fast is the level in the pot rising when the coffee in the cone is 13 cm deep?

b. How fast is the level in the cone falling then?

4. (10 points) Use the **limit definition** to prove that $\lim_{x \rightarrow 1} \sqrt{2x - 1} = 1$.

5. (10 points) Find all asymptotes of the graph of $y = \frac{x^2 - 4}{x - 1}$.

6. (10 points) For all values of the constants m and b for which the function

$$f(x) = \begin{cases} \sin x, & \text{if } x < \pi \\ mx + b, & \text{if } x \geq \pi \end{cases}$$

is

a. (5 points) continuous at $x = \pi$.

b. (5 points) differentiable at $x = \pi$.

(試題結束)