微積分聯合教學　段考一　**Oct 22, 2013**

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。 (不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Let $F(x) = f(g(x))$, $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, $g'(5) = 6$, then $F'(5) = 3$.

2. $\lim_{x\to 5^-} \dfrac{x^2}{(x-5)(3-x)} = \infty$.

3. Let $f(x) = x^3 + x + 1$, then there is no roots for the equation $f(x) = 0$.

4. If $\lim_{x\to 3^-} f(x)$ and $\lim_{x\to 3^+} f(x)$ both exist and equal, then $f(x)$ must continuous at $x = 3$.

5. If L is the line with equation $Ax + By + C = 0$, where $A \neq 0$. Then L crosses the x-axis at the point $(-\frac{C}{A}, 0)$.

6. $(-\infty, -\dfrac{1}{2})$ is the solution of the set $\{x : \dfrac{4x+3}{4x^2+3} \leq 1\}$.

7. The domain of the function $f(x) = \sqrt{2x-3} - \sqrt{6-2x}$ is $\left[\dfrac{3}{2}, 3\right]$.

8. $\lim_{x\to -5} \dfrac{x^2-25}{|x|-5} = 0$.

9. Assume that $\dfrac{d}{dx}F(x) = \dfrac{1}{1+x^2}$, then $\dfrac{d}{dx}(F(x^2))^2 = F(x^2)\dfrac{4x}{1+x^2}$.

10. Suppose $f(1) = 3$, $f'(1) = -2$, $g(1) = 5$, $g'(1) = 7$, and $Q(x) = \dfrac{f(x)}{g(x)}$, then $Q'(1) = -\dfrac{31}{25}$.

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填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $G(y) = \dfrac{(y-1)^4}{(y^2+2y)^5}$, find $G'(y)$.

Answer:_____.

2. A soda can holds 12 fluid ounces (approximately 6.89π cubic inches). Express the lateral (側面的) surface area of the can as a function of its radius.

Answer:_____.

3. Let$f(x) = \begin{cases} x^2+1 & \text{if } x \le 3 \\ 2x+4 & \text{if } x > 3 \end{cases}$, How many discontinuous points are there ?

Answer:_____.

4. Let$f(x) = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$, Find $f'(x)$.

Answer:_____.

5. Find the equation of the line that is perpendicular to the line $2x - 3y = 6$ and passing through the point (2,-3).

Answer:_____.

6. Find the value of L if $\lim\limits_{x\to 2} f(x) = L$ and $\lim\limits_{x\to 2} \dfrac{f^2(x) + xf(x) - \frac{3x}{2}}{\sqrt{f(x)}} = 0$.

Answer:_____.

7. Let $h = f \circ g$; Find $h'(0)$ by the information in the following table:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	2	3	3	2
1	1	−2	0	3
2	3	4	1	0
3	4	−3	2	1

Answer:_____.

8. The monthly demand and supply functions for the Luminar desk lamp are given by $p = d(x) = -1.1x^2 + 1.5x + 40$, $p = s(x) = 0.1x^2 + 0.5x + 15$ respectively, where p is measured in dollars and x in units of a thousand. Find the equilibrium quantity and price.

Answer:_____.

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計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) An efficiency study of the morning shift at a certain factory indicates that an average worker arrising on the job at 8:00 am will have produced $Q(t) = -t^3 + 8t^2 + 15t$ units t hours later.

a. Compute the worker's rate of production?

b. At what rate is the worker's rate of production changing with respect to time at 9:00 am.

2. (*10* points) An urban (城市的) planner models the population $P(t)$ (in thousands) of a centain community t years from now by the function

$$P(t) = \frac{40t}{t^2 + 10} - \frac{50}{t + 1} + 70.$$

a. What is the current population of the community?

b. By how much does the population change during the 3^{rd} year? (第三年人口的瞬時變化率是多少?) Is the population increasing or decreasing over this time period ? (第三年這年的總人口變化是增加還是減少?)

c. What happens to the population in the long run?

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3. (*10* points) A demographic (人口統計學) study models the population P (in thousands) of a community by the equation $P(Q) = 3Q^2 + 4Q + 200$, where Q is a quality-of-life index that ranges from $Q = 0$ (extremly poor quality) to $Q = 10$ (excellent quality). Suppose the index varies with time in such a way that t years from now.

$$Q(t) = \frac{t^2 + 2t + 3}{2t + 1} \qquad for \quad 0 \leq t \leq 10$$

a. What value of the quality-of-life index should be expected 4 years from now? What will be the corresponding population at this time?

b. At what rate is the population changing with respect to time 4 years from now? Is the population increasing or decreasing at this time?

4. (*10* points) Hypocrite Rentals Inc. owns a large apartment complex containing 400 identical apartments. If the monthly rent for each apartment is \$800 all the apartments are rented. However, for each \$60 monthly increase in rent 3 apartments become vacant (空的). Find an expression which may be used to represent their monthly income and determine the monthly rent to be charged to maximize their income.

5. (*10* points) Suppose $f(x)$ is continuous on $[1, 2]$, $f(1) = 0$ and $f(2) = 3$.
Explain the reason why $f(x) = x$ at some point x between 1 and 2.

6. (*10* points) Evaluate the following:

a. $\lim_{x\to\infty}(\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x})$

b. $\lim_{x\to 1} \dfrac{x + x^2 + ... + x^n - n}{x - 1}$

(試題結束)