

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f'(c) = 0$, then f has a local maximum or minimum value at c .
2. If $f''(c) = 0$, then $(c, f(c))$ is a point of inflection.
3. The definite integral $\int_{-\pi}^{\pi} \frac{\sin x \cdot \cos^3 x}{1 + x^4} dx = 0$.
4. $\sum_{k=-1}^4 (2k + 1) = \sum_{k=0}^5 (2k - 1)$.
5. $\int_0^{2\pi} \sin(x) dx = \int_0^{2\pi} \cos(x) dx$.
6. If $\int_0^1 f(x) dx = 4$ and $f(x) \geq 0$, then $\int_0^1 \sqrt{f(x)} dx = 2$.
7. If $f(x)$ is continuous on \mathbb{R} , then $\int_0^x \left(\int_0^u f(t) dt \right) du = \int_0^x f(u)(x - u) du$.
(Hint: Differentiate both sides.)
8. Let $f(x)$ be a continuous function on $[a, b]$ and $F(x) = \int_a^x f(t) dt$ for $x \in [a, b]$.
Then $F(x)$ is differentiable on (a, b) .
9. $\lim_{h \rightarrow 0} \frac{1}{h} \int_x^{x+h} \sqrt{1 + t^2} dt = \sqrt{1 + x^2}$.
10. The Mean Value Theorem can be applied to $f(x) = 1/x$ on the interval $[-1, 1]$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the absolute maximum for $f(x) = x^3 - 6x^2 + 9x - 6$ on the interval $[-1, 5]$.

Answer : _____.

2. Find $\frac{d}{d\theta} \int_0^{\tan \theta} \sec^2 y \, dy$.

Answer : _____.

3. Find the closed interval $[a, b]$ such that $\int_a^b x - x^2 \, dx$ is maximized.

Answer : _____.

4. Find the area enclosed by $y = x^2 + 1$ and $x + y = 3$.

Answer : _____.

5. Find the linear approximation of $f(x) = 5 + \int_1^{x^2} \sec(t - 1) \, dt$ at $x = 1$.

Answer : _____.

6. Evaluate $\int x\sqrt{4-x} \, dx$.

Answer : _____.

7. Find the average value of $f(\theta) = \frac{\tan \theta}{\sqrt{2 \sec \theta}}$ on $[0, \pi/3]$.

Answer : _____.

8. How many roots does the equation $r(\theta) = 2\theta - \cos^2 \theta + \sqrt{2}$ in $(-\infty, \infty)$ have?

Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Show that $1 - \cos x \leq x$ for all $x \geq 0$.
2. (10 points) Evaluate the indefinite integral $\int \sin^2 x \cdot \cos^2 x \, dx$.
3. (10 points) You are designing a rectangular poster to contain 300 cm^2 of printing with a 10 cm margin at the top and bottom and a 5 cm margin at each side. What overall dimensions will minimize the amount of paper used?
4. (10 points) Sketch the graph of $f(x) = \frac{(x+1)^2}{1+x^2}$ by following the steps.
 - a. Find out the domain and symmetries.
 - b. Find out f' and f'' .
 - c. Analyze the behavior of critical points to determine all local maxima and minima.
 - d. Determine increasing and decreasing intervals.
 - e. Determine inflection points and intervals where the graph concaves upward and downward.
 - f. Determine all asymptotes.
 - g. Sketch the graph.
5. (10 points) Find out the length of $y = x^{\frac{3}{2}}$ from $x = 1$ to $x = 2$.
6. (10 points) The region bounded by the curve $y = x^2$, the x -axis and the line $x = 2$ is revolved about the line $x = -1$ to generate the shape of a solid. Compute the volume of the solid.