

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f is continuous on (a, b) , then f has both an absolute maximum value and an absolute minimum value.
2. If the derivative of f at c is equal to zero, then f must have a relative extrema at $x = c$.
3. If $f(x) = \frac{1}{x}$, then 0 is a critical number of f .
4. If $f(x)$ is concave downward on (a, b) , then the graph of f must lie below its tangent lines.
5. If $f(x)$ and $g(x)$ are defined on $[a, b]$, and $f(a) \geq g(a)$ and $f'(x) \geq g'(x)$ for all x in (a, b) , then $f(x) \geq g(x)$ for all x in (a, b) .
6. The total cost of producing the first 250 units of a certain commodity is \$45,500. If marginal cost $C'(250) = 100$, then the total cost of producing the first 251 units is approximately \$45,600.
7. Suppose the elasticity of demand of a certain commodity is $E(p) = \frac{p}{400 - p}$. If the price is \$100, then raising the unit price slightly will cause the revenue to decrease.
8. Suppose $P(t)$ represents the population of bacteria at time t and suppose $P'(t) > 0$ and $P''(t) < 0$, then the population is increasing at time t and at an increasing rate.

9. Let $f(x) = \frac{x}{\sqrt{2x^2 + 1}}$. Then $y = \frac{\sqrt{2}}{2}$ and $y = -\frac{\sqrt{2}}{2}$ are the horizontal asymptotes of the graph of f .
10. If $f''(x) > 0$ on (a, b) and $f'(c) = 0$, where $a < c < b$, then $f(c)$ is the absolute minimum value of f on $[a, b]$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the differential of the function $f(x) = \frac{x+1}{x^2+2}$.
Answer : _____.
2. Let $f(x) = \frac{x+3}{2x^2+3x-2}$. Find all vertical asymptotes and horizontal asymptotes.
Answer : _____.
3. Let a and b be the absolute max and min values of the function $f(x) = x\sqrt{1-x^2}$ on the interval $[-1, 1]$, find $a+b$.
Answer : _____.
4. Find the relative maxima and relative minima, if any, of $f(x) = \frac{x}{x+1}$.
Answer : _____.
5. Find the slope of the tangent line to the graph of the curve $x^2y^3 - y^2 + xy - 1 = 0$ at the point $(1, 1)$.
Answer : _____.
6. The volume of a spherical cancerous tumor is given by $V(r) = \frac{4}{3}\pi r^3$, where r is the radius of the tumor. If the radius is measured with a maximum percentage error of 0.45%, what is the maximum percentage error in the calculated volume of the tumor?
Answer : _____.
7. Determine where the function $f(x) = \frac{1}{7+x^2}$ is concave downward.
Answer : _____.

8. A truck gets $600/x$ mpg when driven at a constant speed of x mph (between 40 and 70 mph). If the price of fuel is \$3/gallon and the driver is paid \$18/hour, at what speed between 40 and 70 mph is it most economical to drive?

Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) The quantity of T-shirts demanded each month is related to the unit price by the equation

$$p(x) = \frac{30}{0.01x^2 + 1} \quad 0 \leq x \leq 30$$

where p is measured in dollars.

- Find the revenue function R .
 - Find the marginal revenue function R' .
 - What is the marginal revenue when $x = 4$.
 - Interpret your result of **c**.
2. (10 points) Let $f(x) = x + \frac{1}{x}$
- Find the intervals where f is increasing and the intervals where f is decreasing.
 - Find the critical points of $f(x)$.
 - Find all possible relative extrema.
3. (10 points) A Norman window has the shape of a rectangle surmounted by a semicircle. If a Norman window is to have a perimeter of 28 ft, what should its dimensions be in order to allow the maximum amount of light through the window?

4. (10 points) Let $f(x) = \frac{x}{x^2 + 1}$. Sketch the graph of f and label its relative extrema and inflection points.
5. (10 points) Suppose the quantity demanded weekly of the Super Titan radial tires is related to its unit price by the equation $p + x^2 = 144$, where p is measured in dollars and x is measured in units of a thousand. How fast is the quantity demanded changing when $x = 9$, $p = 63$, and the price/tire is increasing at the rate of \$2/week?
6. (10 points) The demand equation for the Sicard wristwatch is given by

$$x = 10\sqrt{\frac{50 - p}{p}}, \quad 0 < p \leq 50$$

where x (measured in units of a thousand) is the quantity demanded/week, and p is the unit price in dollars.

- a. Compute the elasticity of demand.
- b. Determine the range of prices corresponding to inelastic, unitary, and elastic demand.

(試題結束)