

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. $\frac{d}{dx}|x|^4 = 4|x|^3$.
2. If $f''(c) = 0$, then $(c, f(c))$ is a point of inflection.
3. $\lim_{h \rightarrow 0} \frac{1}{h} \int_x^{x+h} \sqrt{1+t^2} dt = \sqrt{1+x^2}$.
4. $\int_{-\infty}^{\infty} \frac{4x^3}{1+x^4} dx = \lim_{b \rightarrow \infty} \int_{-b}^b \frac{4x^3}{1+x^4} dx = 0$.
5. $\int_0^2 \frac{dx}{(x-1)^2} = \left[\frac{-1}{x-1} \right]_0^2 = -2$.
6. $\int_1^{\infty} \frac{\sin^2 x}{x^2} dx$ is convergent.
7. If f is continuous on $[0, 1]$, then $\int_0^1 f(x) dx = \int_0^1 f(1-x) dx$.
8. $\cot^{-1}(-1) = -\frac{\pi}{4}$.
9. For any polynomial function $P(x)$, we have $P(x) = O(e^x)$.
10. $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x = e^2$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Order the following functions from slowest growing to fastest growing as $x \rightarrow \infty$.

a. $e^{\frac{x}{2}}$ b. e^x c. x^x d. $(\ln x)^x$

Answer : _____.

2. The region bounded by the curve $y = x^2$, the x -axis and the line $x = 2$ is revolved about the line $x = -1$ to generate the shape of a solid. Compute the volume of the solid. Answer : _____.

3. Evaluate the integral $\int \frac{dx}{x \log_5 x}$.

Answer : _____.

4. Evaluate the integral $\int \sin \sqrt{x} dx$.

Answer : _____.

5. Let $f(x) = x^3 - 2$. Find the value of $\left. \frac{df^{-1}}{dx} \right|_{x=6}$.

Answer : _____.

6. Evaluate the integral $\int_2^3 \frac{2x^3 - 2x^2 + 1}{x^2 - x} dx$.

Answer : _____.

7. Find $f'(2)$ if

$$f(x) = \frac{(x^2 + 1)(x + 3)^{1/2}}{x - 1}, \quad x > 1.$$

Answer : _____.

8. Evaluate $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x dx$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find $\frac{d^2y}{dx^2}$ by **implicit differentiation** if $2x^3 - 3y^2 = 8$.

2. (10 points) Evaluate the integrals: $\int \sec x \, dx$ and $\int \sec^3 x \, dx$.

3. (10 points) Evaluate $\int_0^{\frac{\sqrt{3}}{2}} \frac{4x^2 \, dx}{(1-x^2)^{\frac{3}{2}}}$.

4. (10 points) Find $f'(0)$ for $f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0. \end{cases}$

5. (10 points) Investigate the convergence of $\int_1^{\infty} \frac{x - e^{-x}}{x^2} \, dx$.

6. (10 points) Find the limits.

a. $\lim_{x \rightarrow \infty} (\ln x)^{1/x}$

b. $\lim_{x \rightarrow 0^+} x^x$

(試題結束)