

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. $\sqrt{3}^{\sqrt{27}} = \sqrt{27}^{\sqrt{3}}$.

2. $19^{(\log_5 19)^{-1}} = 5$.

3. $\int_{-1}^1 \frac{1}{x} dx = 0$.

4. $\int_{-5}^5 x^3 e^{-x^2} dx = 0$.

5. If f' is continuous on $[0,1]$, then $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f'(\frac{k}{n}) = f(1) - f(0)$.

6. If $Q_1 = C - Ae^{-kt}$ is a learning curve function and $Q_2 = \frac{A}{1 + Be^{-kt}}$ is a logistic growth function, where $A, C, k > 0$ and $B > 1$, then Q_1' is decreasing whereas Q_2' has a relative maximum.

7. For $x > 0$, let $G(x) = \int_1^x \frac{1}{t} dt$; then $x > 1$ implies $\int_1^x G''(u) du < 0$.

8. The area between the x-axis and the graph of $y = 8 - 3x^2$ for $-2 \leq x \leq 3$ is $\left| \int_{-2}^3 (8 - 3x^2) dx \right|$.

9. The absolute value function does not have an antiderivative on the interval $(-2, 2)$.

10. The producers' surplus is given by $PS = \int_0^{\bar{x}} S(x) dx - \bar{p}\bar{x}$ where $S(x)$ is the supply function, \bar{p} is the unit market price, and \bar{x} is the quantity supplied.

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填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. If $f(x) = \begin{cases} \sqrt{1-x^2} & \text{if } -1 \leq x < 1 \\ -x+1 & \text{if } 1 \leq x \leq 2 \end{cases}$, compute $\int_{-1}^2 f(x)dx$.

Answer : _____.

2. The area A of a healing wound changes at a rate given approximately by $\frac{dA}{dt} = -4t^{-3}$ ($1 \leq t \leq 10$) where t is time in days and $A(1)=2$ square centimeters. What will the area of the wound be in 10 days?

Answer : _____.

3. Evaluate $\int \frac{1}{x(\ln x)(\ln(\ln x))} dx$.

Answer : _____.

4. An average student enrolled in a stenotyping(速記打字) class progressed at a rate of $N'(t) = 12e^{-0.06t}$ words per minute per week t weeks after enrolling in a 15-week course. If, at the beginning of the course, a student could stenotype at zero words per minute, how many words per minute $N(t)$ would the student be expected to handle after completing the course? Answer : _____.

5. Compute the average value of the function $f(x) = \frac{1}{5x}$ over the interval $[1,5]$.

Answer : _____.

6. It is given that $f(x) = Ae^{kx}$, $f(0) = 100$, $f(1) = 120$, find $f(x)$.

Answer : _____.

7. Let $h = f \circ g$. Find $h'(0)$ by the information in the following table:

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 0 | 2 | 3 | 3 | 2 |
| 1 | 1 | -2 | 0 | 3 |
| 2 | 3 | 4 | 1 | 0 |
| 3 | 4 | -3 | 2 | 1 |

Answer : _____.

(下頁還有試題)

8. Find the slope of the tangent line to the graph of the curve $x^2y^3 - y^2 + xy - 1 = 0$ at the point $(1, 1)$.

Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) The number of online buyers in Western Europe is expected to grow steadily in the coming years.

The function

$$P(t) = 28.5 + 14.42 \ln t \quad (1 \leq t \leq 7)$$

gives the estimated online buyers as a percentage of the total population, where t is measured in years, with $t = 1$ corresponding to 2001.

- a. What was the percentage of online buyers in 2001 $t = 1$?
- b. How fast was it changing in 2001?
2. (10 points) The monthly demand for a certain brand of table wine is given by the demand equation

$$p = 240 \left(1 - \frac{3}{3 + e^{-0.0005x}} \right)$$

where p denotes the wholesale price per case (in dollars) and x denotes the number of cases demanded. Find the rate of change of the price per case when $x = 1000$.

3. (10 points) The total accumulated costs $C(t)$ and revenues $R(t)$ (in thousands of dollars), respectively, for a photocopying machine satisfy $C'(t) = \frac{t}{11}$ and $R'(t) = 5te^{-t^2}$ where t is time in years. The value of t for which $C'(t) = R'(t)$ is called the useful life of the machine.
- a. Find the useful life of the machine, to the nearest year. (最接近的整數年)
- b. What is the total profit accumulated during the useful life of the machine ?
($e^4 = 54.598$)

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4. (10 points) Find the Gini index of income distribution for the Lorenz curve with equation $y = \frac{1}{2}x\sqrt{1+3x}$.
5. (10 points) The decay constant for carbon-14 is $k = 0.00012$. A piece of animal bone found at an archaeological site contains 4% of the original amount of carbon-14. Estimate the age of the bone.
6. (10 points) The demand equation for the Sicard wristwatch is given by

$$x = 10\sqrt{\frac{50-p}{p}}, \quad 0 < p \leq 50$$

where x (measured in units of a thousand) is the quantity demanded/week, and p is the unit price in dollars.

- a. Compute the elasticity of demand.
- b. Determine the range of prices corresponding to inelastic, unitary, and elastic demand.

(試題結束)