微積分聯合教學　段考四　**Mar 25, 2014**

考試時間 120 分鐘，題目卷爲兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$ converges conditionally.

2. The curve $r = 1 - \cos\theta$ in polar coordinates is symmetric about the y-axis.

3. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\cos n\pi}{2^n}$ is an alternating series.

4. Let $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ be standard unit vectors, then $|\mathbf{i}+\mathbf{j}+\mathbf{k}| = |\mathbf{i}-\mathbf{j}+\mathbf{k}| = |\mathbf{i}+\mathbf{j}-\mathbf{k}|$.

5. Define $a_0 = 0$ and $a_n = \sqrt{2 + a_{n-1}}, n \geq 1$. Then $\{a_n\}$ is convergent.

6. There exists a power series $\displaystyle\sum_{n=0}^{\infty} c_n (x-a)^n$ whose interval of convergence is $[2, \infty)$.

7. The graphs of $r = -1 + \sin\theta$ and $r = 1 + \sin\theta$ are different.

8. The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges if $p \geq 1$.

9. If $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, and if $a_n > 0$ and $a_n \neq 1$ for all n, then $\displaystyle\sum_{n=1}^{\infty} \frac{a_n}{1-a_n}$ is also convergent.

10. $(0, 5, -5)$ is the point on the sphere $x^2 + (y-3)^2 + (z+5)^2 = 4$ nearest to the xy-plane.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the first four terms of the binomial series for the function

$$f(x) = \left(1 + \frac{x}{3}\right)^{-2}.$$

Answer ： ___________.

2. Find the sum of $\sum_{n=3}^{\infty} \frac{2}{(2n-3)(2n-1)}$.

Answer ： ___________.

3. Evaluate $2 + \ln 2 + \frac{(\ln 2)^2}{2!} + ... + \frac{(\ln 2)^n}{n!} + ...$

Answer ： ___________.

4. Evaluate $\lim_{n\to\infty} \left(\frac{1}{n}\right)^{\frac{1}{\ln n}}$.

Answer ： ___________.

5. Find the point equidistant (at an equal distance) from the points $(0, 0, 0)$, $(0, 4, 0)$, $(3, 0, 0)$ and $(2, 2, -3)$. Answer ： ___________.

6. Find the area under one arch of the cycloid $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$.

Answer ： ___________.

7. Find the value of c if

$$\sum_{n=2}^{\infty} (1 + c)^{-n} = 2.$$

Answer ： ___________.

8. Replace the polar equation

$$r \sin\left(\theta + \frac{\pi}{6}\right) = 2$$

with an equivalent Cartesian equation. Answer ： ___________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the Taylor Series generated by $f(x) = \ln x$ at $x = 2$.

2. (*10* points) Find the surface area generated by revolving the curve $x = \cos t$, $y = 1 + \sin t$, $0 \leq t \leq 2\pi$ about the x-axis.

3. (*10* points) A curve C is defined by the parametric equations $x = t^2$, $y = t^3 - 3t$.

a. Find the equation of the tangent line at $t = 2$.

b. Find $\dfrac{d^2y}{dx^2}$.

4. (*10* points)

a. Find a power series representation for $g(x) = \dfrac{1}{1+x^2}$ and determine the radius of convergence.

b. Find a power series representation for $f(x) = \tan^{-1} x$ by using **a**. What is the radius of convergence of the power series for $\tan^{-1} x$.

5. (*10* points)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$$

a. Find the series' radius and interval of convergence.

b. For what values of x does the series converge absolutely?

c. For what values of x does the series converge conditionally?

6. (*10* points) Determine whether the series is convergent or divergent.

a. $\displaystyle\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n^2}\right)$

b. $\displaystyle\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$

(試題結束)