

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If the region bounded by $y = x^3$, the y -axis, and $y = 1$ in the first quadrant is revolved about the x -axis, then the volume is $V = \pi \int_0^1 (1 - x^3)^2 dx$.
2. Assume $f(x, y)$ has a relative maximum at (a, b) , then $f_x(a, b) = 0 = f_y(a, b)$.
3. Assume $f(x, y)$ is continuous on $(2, \infty)$ and $f(x)$ is decreasing and $\lim_{x \rightarrow \infty} f(x) = 0$ then $\int_2^{\infty} \frac{f(x)}{x} dx$ converges.
4. $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} dx$.
5. In using the Simpson's rule, the number of subintervals n must be even.
6. The area of the region bounded by the x -axis and the graph of the function $f(x) = xe^{-x^2}$ is $\int_{-\infty}^{\infty} xe^{-x^2} dx$.
7. The rate of change of the function $f(x, y) = x^2 - xy^3 + y^2$ in the x -direction at the point $(1, 2)$ is -6 .
8. If (a, b) gives rise to a constrained relative extremum of f subject to the constraint $g(x, y) = 0$, then (a, b) also gives rise to the unconstrained relative extremum of f .

(下頁還有試題)

9. Let $z = 2x^2y + y^3$. Then the actual change in z when x changes from $x = 1$ to $x = 0.98$ and y changes from $y = 2$ to $y = 2.01$ can be approximated by $dz = 4(1)(2)(-0.02) + [2(1) + 3(4)](0.01)$.
10. Given $\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u} + C$,
then $\int \frac{dx}{x^2\sqrt{50-2x^2}} = -\frac{\sqrt{50-2x^2}}{50\sqrt{2}x} + C$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

- Find the first order partial derivative of $z = \sqrt{x^2 + \ln(3x - y^2)}$ with respect to x .
Answer : _____.
- Find the critical points of $z = f(x, y) = e^{(-\frac{1}{2}x^2 + x - y^2)}$.
Answer : _____.
- The demand functions of two commodities are $x = f(p, q) = \frac{q}{1 + \sqrt{p}}$,
 $y = g(p, q) = \frac{3p}{1 + q^4}$. Determine these two are substitute, complementary, or neither.
Answer : _____.
- Evaluate the indefinite integral $\int \frac{x}{(x^2 + 1)\ln(x^2 + 1)} dx$.
Answer : _____.
- Using the trapezoidal rule to estimate $\int_a^b f(x)dx$ with n subintervals, the maximum error incurred is $\frac{M(b-a)^3}{12n^2}$, where M is a number such that $|f''(x)| \leq M$ for $x \in [a, b]$. Find the least bound on error incurred when $\int_{1/2}^1 \ln x dx$ is approximated by the trapezoidal rule with $n = 10$.
Answer : _____.

(下頁還有試題)

6. Evaluate the improper integral $\int_0^{\infty} \frac{xe^{-x^2}}{2 + e^{-x^2}} dx$.

Answer : _____.

7. Find the volume of the solid of revolution obtained by revolving the region bounded by the curves $y = \sqrt{x}$ and $y = \frac{1}{2}x$ about the y -axis.

Answer : _____.

8. Find the maximum and minimum values of the function $f(x, y) = e^{xy}$ subject to the constraint $x^2 + y^2 = 2$.

Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) The productivity of a certain country is given by $f(x, y) = 3x^{\frac{1}{5}}y^{\frac{4}{5}}$ with x units of labor and y units of capital.

a. Find $f_x(x, y)$ and $f_y(x, y)$.

b. If the country is using 243 units of labor and 1024 units of capital.

Find their marginal productivity.

c. Base on the result of **b**, which one should be increased?

2. (10 points) Let $f(x, y) = e^{-(x^2+y^2)}$

a. Find the critical points.

b. Determine the relative extreme of the function.

3. (10 points) Evaluate the following indefinite integrals.

a. $\int \frac{\ln x}{\sqrt{x}} dx$

b. $\int \frac{xe^x}{(x+1)^2} dx$

(下頁還有試題)

4. (10 points) The value of all individual retirement accounts (in trillions of dollars) from 2002 to 2005 is summarized:

Year	2002	2003	2004	2005
Value	2	2.3	2.6	3

- a. Let $x = 2$ denote 2002, find an equation of the least squares line for these data.
- b. Use the results of **a**, to estimate the value in 2006.
5. (10 points) A building in the shape of a rectangular box is to have a volume of 12,000 ft^3 . It is estimated that the annual heating and cooling costs will be \$2/square foot for the top, \$4/square foot for the front and back, and \$3/square foot for the sides.
- a. Find the dimensions of the building that will result in a minimal annual heating and cooling cost.
- b. What is the minimal annual heating and cooling cost?
6. (10 points) Find the maximum percentage error in calculating the surface area of a closed rectangular box if an error of at most 0.2% is made in measuring the length, width, and height of the box.

(試題結束)