微積分聯合教學　段考五　**May 6, 2014**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are orthogonal, then the vectors $\mathbf{u}$ and $\mathbf{v}$ must have the same length.

2. If $f(x,y)$ is a differential function and the point (x_0,y_0) in the domain of f, then $\nabla f(x_0,y_0)$ is normal to the level curve through (x_0,y_0).

3. A particle moves along a smooth curve $\mathbf{r}(t)$, and we use $\mathbf{T}(t)$ to denote $\dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$. If $\mathbf{T}(t)$ is a constant vector, then the particle moves along a straight line.

4. $(\mathbf{i}\times\mathbf{j})\times\mathbf{j}=\mathbf{i}\times(\mathbf{j}\times\mathbf{j})$.

5. If $\mathbf{u}\times\mathbf{v}=\mathbf{u}\times\mathbf{w}$ and $\mathbf{u}\neq 0$, then $\mathbf{v}=\mathbf{w}$.

6. $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{v}=0$.

7. $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^3-xy^2}{x^2+y^2}=0$.

8. There exists a function f with continuous second-order partial derivatives such that $f_x(x,y)=x+y$ and $f_y(x,y)=2x-y$.

9. If $f_x(a,b)$ and $f_y(a,b)$ both exist, then f is differentiable at (a,b).

10. The curvature of a circle of raidus R is $\dfrac{1}{R}$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Express $\dfrac{\partial w}{\partial r}$ in terms of r and s if

$$w = x + 2y + z^2, \quad x = \frac{r}{s}, \quad y = r^2 + \ln s, \quad z = 2r.$$

Answer ：___________.

2. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P(1, 1, 0)$ in the diection of $\mathbf{v} = (2, -3, 6)$.

Answer ：___________.

3. Find the length of the curve $\mathbf{r}(t) = (\sqrt{2}t, \sqrt{2}t, 1-t^2)$ from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$.

Answer ：___________.

4. Find the area of the triangle with vertices $P(1, -1, 0)$, $Q(2, 1, -1)$, and $R(-1, 1, 2)$.

Answer ：___________.

5. Find the distance from $(6, 0, -6)$ to the plane $x - y = 4$.

Answer ：___________.

6. Find $\dfrac{\partial f}{\partial y}$ for $f(x, y) = x^y$.

Answer ：___________.

7. Find the curvature for the helix $\mathbf{r}(t) = (\sin t)\mathbf{i} + (\sqrt{2}\cos t)\mathbf{j} + (\sin t)\mathbf{k}$.

Answer ：___________.

8. Evaluate the double integral $\displaystyle\int\int_R x \cos y \, dA$, where R is the region bounded by $y = 0$, $y = x^2$, and $x = 1$.

Answer ：___________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Let

$$f(x,y) = \begin{cases} \dfrac{2x^2y}{x^4+y^2} & ,\ (x,y) \neq (0,0) \\ 0 & ,\ (x,y) = (0,0) \end{cases},$$

Find the limit of $f(x,y)$ as $(x,y) \to (0,0)$.

2. (*10* points) Sketch the region of integration for the integral

$$\int_0^2 \int_{y^2/4}^{(y+2)/4} (16 - x^2 - y^2)\, dxdy$$

and write an equivalent integral with the order of integration reversed.

3. (*10* points) Find the point on the plane $x+2y+3z = 13$ closest to the point $(1,1,1)$.

4. (*10* points) The derivative of $f(x,y,z)$ at a point P is greatest in the direction of $\mathbf{v} = \mathbf{i} - \mathbf{k}$. In this direction, the value of this derivative is $\sqrt{5}$. Then what is the gradient of f at P? What is the derivative of f at P in the direction of $\mathbf{i} + \mathbf{j}$?

5. (*10* points) Find all critical points of $f(x,y) = xy^2 - x^2 - 2y^2$ and determine whether each is a local maximum, local minimum, or saddle point.

6. (*10* points) Find an equation of the tangent plane and find parametric equations for the normal line to the surface $z = x^2y$ at the point $(2,1,4)$.

(試題結束)