

考試時間 120 分鐘，題目卷為三張紙，共五頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $y = f(x)$ is a solution of a first-order differential equation, then $y = C \cdot f(x)$ is also a solution of the first-order differential equation, where C is any real number.
2. If $y = e^{rt}$ satisfies the differential equation $y'' - y' - 30y = 0$, then the value of r is 6 or -5 .
3. If $I = \iint_D \sqrt{16 - x^2 - y^2} dA$, where D is the disk $\{x^2 + y^2 \leq 16\}$, then $I = \frac{1}{2} \frac{4\pi}{3} 4^3$.
4. For any function f , if $\int_a^b f(x) dx = 1$, then f must be a probability density function on $[a, b]$.
5. The differential equation $y dx - (x + x^2 y) dy = 0$ is separable.
6. A deposit of \$10,000 is made into a bank account and the account earns interest at a rate of 3.2% per year, compounded continuously. The balance, B , in the account in t years satisfies the differential equation $dB/dt = 10,000 + 0.032B$.
7. The median (中位數) of a normal distribution is never at zero.

(下頁還有試題)

8. If $dQ/dt = 5Q - 200$ then when $Q = 10$, the quantity Q will be decreasing.
9. For an iterated integral, one can always switch the order of integration.
10. The waiting time in hours for students in Calculus teacher's office is an exponentially distributed random variable X with associated probability density function $f(x) = \frac{1}{10000}e^{-\frac{x}{10000}}$, then the average waiting time is 10000 hours.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let R be a region bounded by $0 \leq x \leq 1, 0 \leq y \leq 1$.

Then $\iint_R xy e^{x^2} dA = \underline{\hspace{2cm}}$.

Answer : $\underline{\hspace{2cm}}$.

2. Let $f(x) = k \cdot xy e^{-(x^2+y^2)}$ be a joint probability density function on

$D = \{0 < x < \infty, 0 < y < \infty\}$, then $k = \underline{\hspace{2cm}}$.

Answer : $\underline{\hspace{2cm}}$.

3. Let X be a normal random variable with mean 100 and variance 400. If it is given that $P(Z < 1.25) = 0.8944$, then $P(X < 75) = \underline{\hspace{2cm}}$.

Answer : $\underline{\hspace{2cm}}$.

4. Use Euler's method with $n = 4$ to obtain an approximation of the initial value problem $y' = 1 + 2xy, y(0) = 0$ at $x = 1$. Round the answers to the nearest hundredth.

Answer : $y(1) \approx \underline{\hspace{2cm}}$.

5. Suppose that x measures the time (in hours) it takes for a student to complete an exam. All students are done within two hours and the probability density function for x is $f(x) = \frac{x^3}{4}$ if $0 \leq x \leq 2$. What is the average time for students to complete the exam?

Answer : $\underline{\hspace{2cm}}$.

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6. As you know, when a course ends, students start to forget the material they have learned. One model assumes that the rate at which a student forgets material is proportional to the difference between the material currently remembered and some positive constant, a . Let $y = f(t)$ be the fraction of the original material remembered t weeks after the course has ended. Set up a differential equation for y , using k as any constant of proportionality you may need (let $k > 0$) Your equation will contain two constants; the constant A (also positive) is less than y for all t .

Answer : _____.

7. While taking a walk along the road where you live, you accidentally drop your glove, but you don't know where. The probability density function $f(x)$ for having dropped the glove x kilometers from home (along the road) is $f(x) = 2e^{-2x}$ for $x \geq 0$. What is the probability that you dropped it within 1 kilometer of home?

Answer : _____.

8. Suppose that Congress enacts (國會制定) a one-time-only 10% tax rebate (折扣) that is expected to infuse (注入) \$ y billion, $5 \leq y \leq 7$, into the economy. If every person and every corporation is expected to spend a proportion x , $0.6 \leq x \leq 0.8$, of each dollar received, then, by the multiplier principle in economics, the total amount of spending S (in billions of dollars) generated by this tax rebate is given by $S(x, y) = \frac{y}{1-x}$. What is the average total amount of spending for the indicated ranges of the values of x and y ?

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Evaluate $\int_0^2 \int_{x^2}^4 xe^{y^2} dydx$.

2. (10 points) The resale value of a certain machine decreases at a rate proportional to the machine's current value. The machine was purchased at \$50,000 and 2 yr later was worth \$32,000.

a. Find an expression for the resale value of the machine at any time t .

b. Find the value of the machine after 5 yr.

3. (10 points) The population density of a certain city is given by the function

$$f(x, y) = \frac{50,000|xy|}{(x^2 + 20)(y^2 + 36)}$$

where the origin $(0, 0)$ gives the location of the government center. Find the population inside the rectangular area described by

$$R = \{(x, y) | -15 \leq x \leq 15; -20 \leq y \leq 20\}.$$

4. (10 points) The scores on an economics examination are normally distributed with a mean of 72 and a standard deviation of 16. If the instructor assigns a grade of A to top 10% of the class, what is the approximate lowest score a student may have and still

obtain an A? ($\int_0^{1.28} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \approx 0.4$)

(下頁還有試題)

5. (10 points) The life span (in years) of a certain make of car battery is an exponentially distributed random variable with an expected value of 5. Find the probability that the life span of a battery is
- more than 6 yr.
 - between 2 and 4 yr.
6. (10 points) A container that has a constant cross section A is filled with water to height H . The water is discharged through an opening of cross section B at the base of the container. By using Torricelli's Law, it can be shown that the height h of the water at time t satisfies the initial-value problem (1 ft = 12 in)

$$\frac{dh}{dt} = -\frac{B}{A}\sqrt{2gh} \qquad h(0) = H$$

- Find an expression for h .
- Find the time T it takes for the tank to empty.
- Find T if $A=4(\text{ft}^2)$, $B=1(\text{in.}^2)$, $H = 16(\text{ft})$, and $g=32(\text{ft}/\text{sec}^2)$.

(試題結束)