微積分聯合教學　段考六　**Jun 17, 2014**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f(x, y)$ is a differential function and the point (x_0, y_0) in the domain of f, then $\nabla f(x_0, y_0)$ is normal to the level curve through (x_0, y_0).

2. If $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ and $\mathbf{u} \neq 0$, then $\mathbf{v} = \mathbf{w}$.

3. There exists a function f with continuous second-order partial derivatives such that $f_x(x, y) = x + y$ and $f_y(x, y) = 2x - y$.

4. If $\mathbf{F}$ is a force field then the work done by $\mathbf{F}$ as it moves a object around a circle is 0.

5. If $f(x, y, z)$ is continuous on $\mathbf{R}^3$ then

$$\int_0^1 \int_0^1 \int_{x^2}^1 f(x, y, z)dydxdz = \int_0^1 \int_{y^2}^1 \int_0^1 f(x, y, z)dydxdz.$$

6. The integral $\displaystyle\int_0^{\pi} \int_0^{2\sin\theta} \int_0^{r^2} dzdrd\theta$ represents the volume enclosed by $z = 0$, $x^2 + (y-1)^2 = 1$ and $z = x^2 + y^2$.

7. For a graph $z = f(x, y)$ over a region R in the xy-plane, the surface area is $\displaystyle\int\int_R \sqrt{1 + {f_x}^2 + {f_y}^2}\, dxdy$.

8. $(x - 3z)\mathbf{j} - y\mathbf{k}$ is the curl of $\mathbf{F} = xz\mathbf{i} + xy\mathbf{j} + 3xz\mathbf{k}$.

(下頁還有試題)

9. The line integral of the constant function $f(x, y, z) = 10$ over a smooth curve C of length 5 is 50.

10. If f has continuous partial derivatives on $\mathbf{R}^3$ and C is any circle, then $\int_C \nabla f \cdot d\mathbf{r} = 0$.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find an equation of the tangent plane to the surface $z = x^2 y$ at the point $(2, 1, 4)$. Answer ：___________.

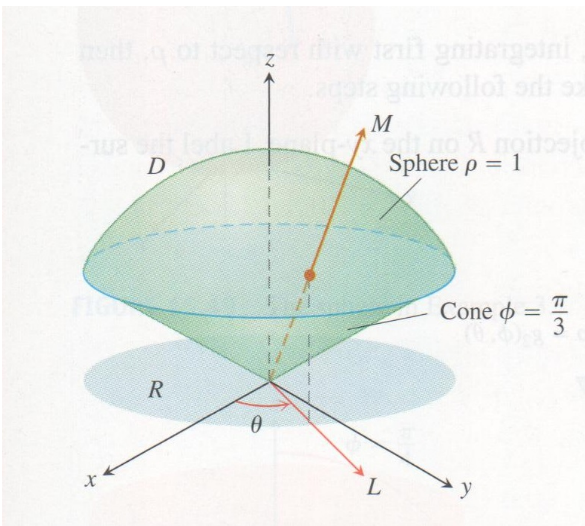
2. Find the work done by the force $\mathbf{F} = y\mathbf{i} - x\mathbf{j}$ over the straight line from $(1, 0)$ to $(0, 1)$. Answer ：___________.

3. Find the circulation of the field $\mathbf{F} = (x^2 - y)\mathbf{i} + 4z\mathbf{j} + x^2\mathbf{k}$ around the curve C in which the plane $z = 2$ meets the cone $z = \sqrt{x^2 + y^2}$, counterclockwise as viewed from above. Answer ：___________.

4. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)\,dydx$.
Answer ：___________.

5. Convert the integral $\int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_0^{x} (x^2 + y^2)\, dzdxdy$ to an equivalent integral in cylindrical coordinates and evaluate the result. Answer ：___________.

6. Set up the integral with the order $d\rho d\phi d\theta$ for evaluating the volume of the "ice cream cone" cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$.
(**Do not evaluate the integral**) Answer ：___________.



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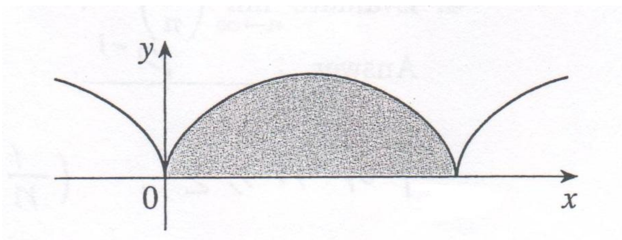
7. Find the center of mass of a thin plate of density $\delta = 3$ bounded by the lines $x = 0$, $y = x$, and the parabola $y = 2 - x^2$ in the first quadrant.

Answer ：___________.

8. Find the area of the portion of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 4$. Answer ：___________.

計算問答證明題 **Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the area under one arch of the cycloid $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$.



2. (*10* points) Evaluate the integral $\displaystyle\int_1^2 \int_{1/y}^{y} \sqrt{\frac{y}{x}} e^{\sqrt{xy}}\, dx dy.$

(Hint: By applying the transformation $u = \sqrt{xy}$ and $v = \sqrt{\dfrac{y}{x}}$.)

3. (*10* points) Apply Green's Theorem to evaluate the integral

$$\oint_C (6y + x)dx + (y + 2x)dy,$$

where C : The circle $(x-2)^2 + (y-3)^2 = 4$.

4. (*10* points) Find the work done by the force field $\mathbf{F} = \dfrac{x\mathbf{i} + y\mathbf{j}}{(x^2+y^2)^{3/2}}$ over the plane curve $\mathbf{r}(t) = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j}$ from the point $(1, 0)$ to the point $(e^{2\pi}, 0)$.

5. (*10* points) Show that $\mathbf{F} = (e^x \cos y + yz)\mathbf{i} + (xz - e^x \sin y)\mathbf{j} + (xy + z)\mathbf{k}$ is conservative over its natural domain and find a potential function for it.

(試題結束)