

考試時間 120 分鐘，題目卷為二張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f is a polynomial of degree five and $P_5(x)$ is the fifth Taylor polynomial of f at $x = 3$, then $f(x) = P_5(x)$ for every value of x .
2. Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all n , then $\sum a_n$ is convergent.
3. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$.
4. $\sum_{n=0}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$.
5. Suppose that $\sum_{n=0}^{\infty} a_n x^n$ has radius of convergence 2.
Then $\sum_{n=0}^{\infty} a_n^2 \left(\frac{5}{2}\right)^n$ converges.
6. Suppose the Taylor series of $f(x)$ at $x = 1$ is $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1}$.
Then $f^{(5)}(1) = 24$.
7. The radius of convergence of $\sum_{n=0}^{\infty} a_n (x-a)^n$ is $\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|}$.
8. $\tan x$ is increasing in $(-\frac{\pi}{2}, \frac{\pi}{2})$.

(下頁還有試題)

9. $\int \tan x dx = \ln |\sec x| + C.$

10. $\lim_{x \rightarrow 0^+} \frac{\sin x - 1}{x} = \lim_{x \rightarrow 0^+} \frac{\cos x}{1} = 1.$

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Express $\frac{1}{(1-x)^2}$ as a power series and find its radius of convergence.

Answer : _____.

2. Find an equation of the tangent line to the graph of the function

$$f(x) = (\tan x)^2 + e^x \text{ at } (0, 1).$$

Answer : _____.

3. Determine whether the series $\sum_{n=1}^{\infty} \left[\frac{1}{3^n} - \frac{1}{n(n+1)} \right]$ converges or diverges.

If it converges, find its sum.

Answer : _____.

4. Find the radius of convergence and the interval of convergence of the power

$$\text{series } \sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+1)3^n}.$$

Answer : _____.

5. Use Newton's method to approximate $\sqrt[3]{2}$. Start the iteration with $f(x) = x^3 - 2$

and initial guess $x_0 = 1$. What is the value of x_1 ?

Answer : _____.

6. Find $\int x \sec^2 x dx.$

Answer : _____.

(下頁還有試題)

7. Find the maximum and minimum values of the function $f(x, y) = e^{xy}$ subject to the constraint $x^2 + y^2 = 4$.

Answer : _____.

8. Suppose that x measures the time (in hours) it takes for a student to complete an exam. All students are done within two hours and the probability density function for x is $f(x) = \frac{x^3}{4}$ if $0 \leq x \leq 2$. What is the average time for students to complete the exam?

Answer : _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

- (10 points) Evaluate $\lim_{n \rightarrow \infty} \left(\frac{n+1}{n} \right)^n$.
(Hint: use L'Hôpital's Rule, let $y = \left(\frac{n+1}{n} \right)^n$ and take \ln)
- (10 points) Find the 3rd Taylor polynomial of $f(x) = \tan x$ at $x = 0$.
- (10 points) Determine whether the series is convergent or divergent.

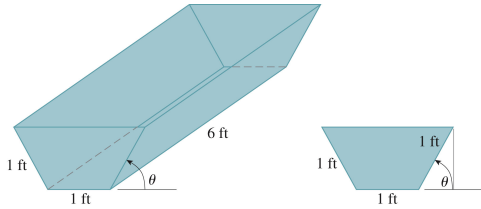
Give your reason.

- $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$
- $\sum_{n=1}^{\infty} \frac{e^{2/\sqrt{n}}}{n^2 + 1}$

- (10 points) Suppose that the rate of air flow in and out a person's lungs during respiration is $R(t) = \frac{3}{5} \sin\left(\frac{\pi}{3}t\right)$ liters per second, where t is the time in seconds. Find an expression for the volume of air V in the person's lungs at any time t . Assume $V(0) = 0$.

(下頁還有試題)

5. (10 points) A trough with a trapezoidal cross section is to be constructed with a 1-foot base and sides that are 6 feet long and 1 foot wide, as shown in the figure. Find the angle of inclination θ that maximizes the capacity of the trough.



6. (10 points) The resale value of a certain machine decreases at a rate proportional to the machine's current value. The machine was purchased at \$50,000 and 2 yr later was worth \$32,000.
- Find an expression for the resale value of the machine at any time t .
 - Find the value of the machine after 5 yr.

(試題結束)