

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。  
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. There exists a function  $y = f(x)$  which has three horizontal asymptotes.
2. The domain of  $f(x) = \frac{x+2}{x^2-x-6}$  is  $(-\infty, 3) \cup (3, \infty)$ .
3.  $y = \sin(x) + x^2 - 5$  has a root between 0 and 1.
4.  $\lim_{x \rightarrow \infty} f(x) = 3$  if for any positive  $\epsilon > 0$  there is an  $M > 0$  such that whenever  $x > M$  one has  $|f(x) - 3| < \epsilon$ .
5. If  $f(x)$  is an odd function and  $f'(c) = 5$ , then  $f'(-c) = -5$ .
6. If  $\lim_{x \rightarrow 0} f(x^3)$  exists, then  $\lim_{x \rightarrow 0} f(x)$  exists and  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} f(x^3)$ .
7. The graph of
$$g(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$
has a tangent line at the origin  $(0, 0)$ .
8. If  $f(x)$  is not differentiable at  $x = 2$ , then it is not continuous at  $x = 2$ .

(下頁還有試題)

9.  $\lim_{x \rightarrow 2^-} \frac{x-3}{x^2-4} = \infty.$

10. If  $f(x) > g(x)$  for all  $x$  near  $a$  (except possibly at  $a$ ), and both  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  exist, then  $\lim_{x \rightarrow a} f(x) > \lim_{x \rightarrow a} g(x).$

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Evaluate  $\lim_{t \rightarrow -\infty} \frac{2-t+\sin t}{t+\cos t}.$

Answer : \_\_\_\_\_.

2. Let  $f(x) = x^2 + [x^5 + (x^3 + 2)^8]^{12}.$  Find  $f'(-1).$

Answer : \_\_\_\_\_.

3. Find  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x}).$

Answer : \_\_\_\_\_.

4. Find  $\lim_{x \rightarrow 3^-} \frac{\lfloor x \rfloor}{x}.$

Answer : \_\_\_\_\_.

5. Find the linearization for  $f(x) = x^{\frac{2}{3}}$  at 27.

Answer : \_\_\_\_\_.

6. Let  $f(x) = \frac{1}{1 + \tan x}.$  Find  $f'(x).$

Answer : \_\_\_\_\_.

7. Find  $\lim_{x \rightarrow 0^+} \left[ \left( \sin \frac{1}{x} \right) (\cos x) \right].$

Answer : \_\_\_\_\_.

8. Find an equation of the normal to the curve  $y = x^3 - 4x + 1$  at the point  $(2, 1).$

Answer : \_\_\_\_\_.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Use  $\epsilon - \delta$  argument to prove  $\lim_{x \rightarrow 1} f(x) = 1$  if

$$f(x) = \begin{cases} x^2, & \text{if } x \neq 1. \\ 2, & \text{if } x = 1. \end{cases}$$

2. (10 points) Find an equation of the tangent line to the curve  $x^2 \cos^2 y - \sin y = 0$  at  $(0, \pi)$ .
3. (10 points) Water runs into a conical tank at the rate of  $9 \text{ ft}^3/\text{min}$ . The tank stands point down and has a height of 10 ft and a base radius of 5 ft. How fast is the water level rising when the water is 6 ft deep?
4. (10 points) Find the derivative  $\frac{dy}{dt}$  if  $y = \tan^3(\sin^4(5t))$ .
5. (10 points) For all values of the constants  $a$  and  $b$  for which the function

$$f(x) = \begin{cases} \frac{2 \sin^2 x}{x}, & \text{if } x > 0 \\ ax + b \cos x, & \text{if } x \leq 0 \end{cases}$$

is

- a. continuous at  $x = 0$ .
- b. differentiable at  $x = 0$ .
6. (10 points) Find all asymptotes of the graph of  $y = \frac{x(x+5)}{x-4}$ .

(試題結束)