

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If  $f(x)$  is NOT continuous at  $a$ , then  $f(x)$  is NOT differentiable at  $a$ .
2. Suppose that  $L_1$  and  $L_2$  are two lines with slopes  $m_1 \neq 0$  and  $m_2 \neq 0$  respectively. Is the following true?  $L_1$  and  $L_2$  are perpendicular if and only if  $m_1 m_2 = -1$ .
3.  $\lim_{x \rightarrow \infty} \frac{x^3 + 2x^2 - 8x + 3}{4x^3 - 100x + 3000} = 2$ .
4. If  $h(x) = f(x)/g(x)$ , then  $h'(x) = f'(x)/g'(x)$ .
5.  $x^3 - 10x + 9 = 0$  has a root.
6. The domain of the function  $\frac{1}{\sqrt[3]{2x+3}} + \sqrt{6-4x}$  is  $(-1.5, 1.5]$ .
7. If  $a = -\frac{1}{2}$ ,  $b = \frac{5}{2}$  then the function  $f(x) = \begin{cases} 2 & \text{if } x \leq -1 \\ ax^2 + b & \text{if } -1 < x < 3 \\ -2 & \text{if } x \geq 3 \end{cases}$  is continuous on the entire real line.
8. If  $\left| \lim_{x \rightarrow a^+} f(x) \right| + \left| \lim_{x \rightarrow a^-} f(x) \right| = 0$ , then  $\lim_{x \rightarrow a} f(x)$  exists.
9. Your annual salary was \$26,300 in 2002 and \$29,700 in 2004. Assume your salary can be modeled by a linear equation. Then your salary in 2015 will be \$48,400.
10.  $\lim_{x \rightarrow 0^+} \frac{x^2 + x}{|x|} = \lim_{x \rightarrow 0^-} \frac{x^2 + x}{|x|} = 1$ .  
(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. The demand function for a product is given by

$$p = d(x) = x^2 - 12x + 35 \quad (0 \leq x \leq 5)$$

and the corresponding supply function is given by

$$p = s(x) = x^2 + 15 \quad 0 \leq x \leq 5$$

where  $d$  and  $s$  are in dollars and  $x$  is measured in units of a thousand. Find the equilibrium quantity and price. Answer by putting them in an ordered pair (equilibrium quantity, equilibrium price).

Answer: \_\_\_\_\_.

2. Suppose that  $\lim_{x \rightarrow 5} f(x) = 3$ ,  $\lim_{x \rightarrow 5} g(x) = 2$  and  $\lim_{x \rightarrow 5} h(x) = -1$ . Compute

$$\lim_{x \rightarrow 5} \frac{x^2 + [f(x)]^2}{g(x) + 1}.$$

Answer: \_\_\_\_\_.

3. Let  $f(x) = \begin{cases} x^2/(x-3) & \text{if } x \geq 2; \\ x & \text{if } x < 2. \end{cases}$  Indicate ALL the points where  $f(x)$  is not continuous at. **You can get credits only if a complete answer is given.**

Answer: \_\_\_\_\_.

4. If  $g(x) = \frac{1}{1+x^2} + \sqrt[3]{x}$ , then find  $g(g(-1))$ .

Answer: \_\_\_\_\_.

5. A stone is thrown upwards; its height  $t$  seconds later is  $50t - 16t^2$  feet. Find the velocity of the stone when it hits the ground.

Answer: \_\_\_\_\_ ft/s.

6. Find the limit  $\lim_{x \rightarrow 0} \frac{\sqrt{3x^2 + 1} - 1}{x^2}$ .

Answer: \_\_\_\_\_.

(下頁還有試題)

7. If  $f(x) = \sqrt{2x + \sqrt{3x + \sqrt{4x}}}$ , then find  $f'(1)$ .

Answer: \_\_\_\_\_.

8. The inventor of a new game believes that the variable cost for producing the game is \$0.95 per unit. The fixed cost is \$6000. The selling price for each game is \$1.69. How many units must be sold before the average cost per unit falls below the selling price?

Answer: \_\_\_\_\_.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) A 2014 Chevrolet Malibu costs \$22,340 with a gasoline engine. A 2014 Toyota Prius costs \$24,200 with a hybrid engine. The Malibu gets 16 miles per gallon of gasoline and the Prius gets 35 miles per gallon of gasoline. Assume that the price of gasoline is \$4. The cost of driving the Chevrolet Malibu  $x$  miles is then  $f(x) = 22,340 + \frac{x}{4}$ .

a. Find the cost of driving the Toyota Prius as a function of  $x$ .

b. Find the break-even point. That is, find the mileage  $x$  at which the hybrid-powered Toyota Prius becomes more economical than the gasoline-powered Chevrolet Malibu. (mileage : 哩程數)

2. (10 points) Suppose the life expectancy of a male at birth in a certain country is described by the function  $f(t) = 46.9(1 + 1.09t)^{0.1}$  ( $0 \leq t \leq 150$ ) where  $t$  is measured in years, with  $t = 0$  corresponding to the beginning of 1900.

a. How long can a male born at the beginning of 2000 in that country expect to live?

b. What is the rate of change of the life expectancy of a male born in that country at the beginning of 2000?

(下頁還有試題)

3. (10 points) Let  $y = x^3 - 4x$ .
- Find the equation of the line tangent to this curve at  $x = 1$ , which is midway between two successive roots  $x = 0$  and  $x = 2$ .
  - Find the  $x$ -intercept of the tangent line obtained in (a.), that is, the point where the line crosses the  $x$ -axis.
  - Find the roots of  $x^3 - 4x = 0$ . Do you see any relationship between these roots and the intercept of the tangent line?
4. (10 points) Suppose that the number of riders per day on the New York City subway system is 4 (million) when the fare is \$2.00. Suppose that the ridership will drop to 3.8 (million) when the fare is raised to \$2.25. Assuming that the relationship between demand and price is linear:
- Find the demand equation.
  - Find the revenue and marginal revenue functions.
  - Find the price and number of riders that will maximize the total revenue.
5. (10 points) The demand and cost functions for a product can be modeled by  $p = 211 - 0.002x$  dollars and  $C = 30x + 1500000$  dollars where  $x$  is the number of units produced.
- Write the profit function for this product.
  - Find the marginal profit when 80,000 units are produced.
6. (10 points) Consider the demand equation

$$x = -2p^2 + 800 \quad (0 \leq p \leq 20).$$

- Find the elasticity of demand  $E(p)$ .
- Is demand elastic, unitary or inelastic when  $p=10$ ? When  $p=15$ ?
- If the price is 10, will raise the unit price slightly cause the revenue to increase or decrease?

(試題結束)