

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Suppose that f has a positive derivative for all values of x , and $f(1) = 0$.
 $g(x) = \int_0^x f(t) dt$, then the graph of g has an inflection point at $x = 1$.
2. $\int_{-1}^1 2x^6 dx = \int_{-1}^1 x^5(2x dx) = \int_{-1}^1 (x^2)^{5/2}d(x^2) = \int_1^1 u^{5/2}du$.
3. For $-\frac{\pi}{2} < a < b < \frac{\pi}{2}$, $|\tan a - \tan b| \geq |\sin a - \sin b|$.
4. If $a < b < c$ and f is integrable on $[a, c]$, then $\int_a^c f(x) dx + \int_c^b f(x) dx = \int_a^b f(x) dx$.
5. Let f be a function on $[0, 1]$ and $c \in (0, 1)$. If f has a local maximum at $x = c$, then $f'(c) = 0$.
6. The function $f(x) = (x^2 - 1)^{1/2}$ has three critical points.
7. If $f'(x)$ is continuous on $[0, 1]$ and $f(0) = 0$, then $\int_0^\alpha \sqrt{1 + [f'(x)]^2} dx \geq \sqrt{\alpha^2 + [f(\alpha)]^2}$ for $0 \leq \alpha \leq 1$.
8. If $P(c, f(c))$ is an inflection point of the curve $y = f(x)$, then $f''(c) = 0$.
9. If $f'(x)$ exists and is nonzero for all x , then $f(1) \neq f(0)$.
10. The definite integral $\int_{-\pi/8}^{\pi/8} \frac{x^2 \sin(x)}{1 + x^6} dx = 0$.
(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Use integral to approximate $\sum_{i=1}^{10000} \sqrt{i}$. (Set $\Delta x = \frac{1}{10000}$, $a = 0$, $b = 1$.)

Answer : _____.

2. The area of the region in the xy -plane enclosed by the x -axis, the curve $y = f(x)$, $f(x) \geq 0$ and the lines $x = 1$ and $x = b$ is equal to $\sqrt[3]{b^2 + 1} - \sqrt[3]{2}$ for all $b > 1$. Find $f(x)$.

Answer : _____.

3. Let A denote the area between the graph of $f(x) = \cos x$ and the x -axis over $[0, 2\pi]$, Find A .

Answer : _____.

4. Find $\frac{d}{dx} \int_0^{\sqrt{x}} \cos t \, dt$.

Answer : _____.

5. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin x}{x}$, $x > 0$. Express $\int_1^3 \frac{\sin 2x}{x} \, dx$ in terms of F .

Answer : _____.

6. Let $f(x) = x^{1/4}(2-x)^{3/4}$. Find the absolute maximum value of $f(x)$ on $[0, 1]$.

Answer : _____.

7. Find the average value of $f(x) = x - 2|x|$ on $[-1, 2]$.

Answer : _____.

8. Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for $f(x) = x + \frac{1}{x}$ in $\left[\frac{1}{3}, 3\right]$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Let $f(x) = \frac{2x^2}{x^2 - 1}$. Find **a.** its domain, **b.** critical numbers, **c.** intervals of increasing/decreasing, **d.** local maximum/minimum values, **e.** intervals of concavity, **f.** inflection points, **g.** oblique, horizontal and vertical asymptotes. **h.** Then sketch the graph of $f(x)$.
2. (10 points)
 - a. Find $\int_{-1}^{-\frac{1}{2}} t^{-2} \sin^2 \left(1 + \frac{1}{t}\right) dt$.
 - b. Find the area enclosed by the curves $x + y^2 = 3$ and $4x + y^2 = 0$.
3. (10 points) Let $f(x) = |\sin |x||$, find $f''(x)$.
4. (10 points) **Design a poster** You are designing a rectangular poster to contain 300 cm^2 of printing with a 10 cm margin at the top and bottom and a 5 cm margin at each side. What overall dimensions will minimize the amount of paper used?
5. (10 points) The region bounded by the curve $y = \sqrt{x}$, the x -axis, and the line $x = 4$ is revolved about the x -axis to generate a solid.
 - a. Find the volume of the solid by the disk method.
 - b. Find the volume of the solid by the shell method.
6. (10 points) Let $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \leq x \leq 4$.
 - a. Find the length of the graph of f .
 - b. Find the area of the surface generated by revolving the curve $y = f(x)$ about the x -axis.

(試題結束)