

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Let  $y = \sqrt{144 - x}$ . The actual change in  $y$ ,  $\Delta y$ , as  $x$  changes from 108 to 107.9 can be approximated by  $\frac{1}{120}$ .
2. The relative maximum of  $f(x) = x + \frac{1}{x}$  is smaller than its relative minimum.
3. If  $f''(3) = 0$ , then  $(3, f(3))$  is an inflection point of  $f$ .
4. If  $f$  is continuous on  $(a, b)$ , then  $f$  has an absolute maximum  $f(c)$  and an absolute minimum  $f(d)$  for some numbers  $c$  and  $d$  in  $(a, b)$ .
5. The line  $x = -1$  is a vertical asymptote of the graph of  $f(x) = \frac{x^2 - 1}{x(x + 1)}$ .
6. Any real numbers  $x$  which make  $f'(x) = 0$  or  $f'(x)$  not exist are called critical numbers of the function  $f$ .
7. A polynomial functions of degree  $n$  ( $n > 2$ ) can have at most  $(n - 1)$  inflection points.
8. The function  $f(x) = \ln|x|$  is continuous for all  $x \neq 0$ .
9. There is no way to find the derivative of  $\frac{dy}{dx}$  if the relation between  $x$  and  $y$  is not in the form of  $y = f(x)$ .
10.  $\frac{\ln 20}{\ln 7} = \ln \frac{20}{7}$ .

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let  $f(x) = \frac{x}{x+1}$ . Find  $f^{(3)}(1)$ .

Answer: \_\_\_\_\_.

2. In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the area polluted is circular, determine how fast the area is increasing when the radius of the circle is 60 ft and is increasing at the rate of  $\frac{1}{2}$  ft/sec.

Answer: \_\_\_\_\_.

3. Suppose the side of a cube is measured with a maximum percentage error of 0.1%. Use differentials to estimate the maximum percentage error in the calculated surface area of the cube.

Answer: \_\_\_\_\_.

4. Find the absolute extrema of  $f(x) = x - 4\sqrt{x}$  on the interval  $[0, 9]$ .

Answer: \_\_\_\_\_.

5. If  $f(x) = Axe^{-kx}$ , find  $f(3)$  if  $f(1) = 3$  and  $f(2) = 5$ .

Answer: \_\_\_\_\_ ft/s.

6. The height (in feet) of a certain kind of tree is approximated by

$$h(t) = \frac{160}{1 + 240e^{-0.2t}},$$

where  $t$  is the age of the tree in years. Estimate the age of an 80-ft tree.

Answer: \_\_\_\_\_.

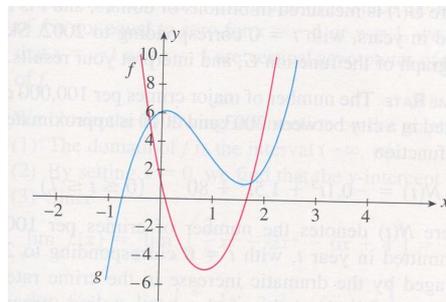
7. Simplify the function

$$\ln \left[ \frac{x^{11} \sqrt[8]{6 - x^5}}{e^{2x}} \right]$$

to the simplicity form. Answer: \_\_\_\_\_.

(下頁還有試題)

8. You are given the graphs of two functions  $f$  and  $g$ . One function is the derivative function of the other. Identify each of them.



Answer: \_\_\_\_\_.

計算問答證明題 **Please show all your work** (60 points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (10 points) The demand function for a certain brand of compact disc is  $p = -0.01x^2 - 0.2x + 8$ , where  $p$  is the wholesale unit price in dollars and  $x$  is the quantity demanded each week, measured in units of a thousand. Compute the elasticity of demand and determine whether the demand is inelastic, unitary, or elastic when  $x = 15$ . (Hint: Given demand function  $x = f(p)$ , the elasticity of demand at  $p$  is  $E(p) = -pf'(p)/f(p)$ .)
2. (10 points) Find the asymptotes, relative extrema, and inflection points, if any, of  $f(x) = \frac{x}{\sqrt{x^2 + 1}}$ . Then sketch the graph of  $f$ .
3. (10 points) Neilsen Cookie Company sells its assorted butter cookies in containers that have a net content of 1 lb. The estimated demand for the cookies is 1,000,000 1-lb containers per year. The setup cost for each production run is \$500, and the manufacturing cost is \$0.50 for each container of cookies. The cost of storing each container of cookies over the year is \$0.40. Assume uniformity of demand throughout the year and instantaneous production.
  - a. Find the total cost function.
  - b. How many containers of cookies should Neilsen produce per production run in order to minimize the total cost?

(下頁還有試題)

4. (10 points) Jane took 500mg of a drug in the morning and another 50mg of the same drug at the same time the following morning. The amount of the drug in her body  $t$  days after the first dose was taken is given by

$$A(t) = \begin{cases} 50e^{-t} & \text{if } 0 \leq t < 1 \\ 50(1 + e^{-1})e^{-t} & \text{if } t \geq 1 \end{cases}$$

- a. What was the amount of drug in Jane's body immediately after taking the second dose?
- b. What was the amount of drug in Jane's body in the long run?
5. (10 points) A car leaves an intersection traveling west. Its position 4 sec later is 20 ft from the intersection. At the same time, another car leaves the same intersection heading north so that its position  $t$  sec later is  $t^2 + 2t$  ft from the intersection. If the speed of the first car 4 sec after leaving the intersection is 9 ft/sec, find the rate at which the distance between the two cars is changing at the instant of time.
6. (10 points) Use differential to approximate

$$\sqrt[4]{256.31} + \frac{1}{\sqrt[4]{255.99}}.$$

(試題結束)