

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The domain of  $f(x) = \frac{x+2}{x^2-x-6}$  is  $(-\infty, 3) \cup (3, \infty)$ .

2. If  $\lim_{x \rightarrow 0} f(x^3)$  exists, then  $\lim_{x \rightarrow 0} f(x)$  exists and  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} f(x^3)$ .

3. The function  $f(x) = (x^2 - 1)^{1/2}$  has three critical points.

4. Since  $\lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2+1} dx = 0$ , we have

$$\int_{-\infty}^{\infty} \frac{2x}{x^2+1} dx = \lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2+1} dx = 0.$$

5. Functions that are neither increasing nor decreasing may still be one-to-one and have an inverse.

6.  $\frac{d}{dx} 3^{\sin x} = 3^{\sin x} \ln 3$ .

7.  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x + x^2} = \lim_{x \rightarrow 0} \frac{\sin x}{1 + 2x} = \lim_{x \rightarrow 0} \frac{\cos x}{2} = \frac{1}{2}$ .

8.  $\int_0^3 \frac{dx}{x-1} = \ln|x-1| \Big|_0^3 = \ln 2 - \ln 1 = \ln 2$ .

9. The inverse function of  $e^{2\sqrt{x}}$  is  $y = \frac{1}{4}(\ln x)^2$ ,  $x > 1$ .

10.  $x^4 + \ln x = O(x^4)$  and  $x^4 = O(x^4 + \ln x)$ .

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x})$ .

Answer : \_\_\_\_\_.

2. Order(排序) the following functions from slowest growing to fastest growing as

$x \rightarrow \infty$ .    **a.**  $e^{\frac{x}{2}}$     **b.**  $e^x$     **c.**  $x^x$     **d.**  $(\ln x)^x$

Answer : \_\_\_\_\_.

3. Find a value of  $c$  that makes the function

$$f(x) = \begin{cases} \frac{9x - 3 \sin 3x}{5x^3}, & x \neq 0 \\ c, & x = 0 \end{cases}$$

continuous at  $x = 0$ .

Answer : \_\_\_\_\_.

4. Evaluate  $\int_1^e x(\ln x)^2 dx$ .

Answer : \_\_\_\_\_.

5. Let  $f(x) = x^3 - 3x^2 - 1$ ,  $x \geq 2$ . Find  $\frac{df^{-1}}{dx} \Big|_{x=-1}$ .

Answer : \_\_\_\_\_.

6. Find  $\lim_{x \rightarrow \infty} (1 + 2x)^{1/(2 \ln x)}$ .

Answer : \_\_\_\_\_.

7. Evaluate  $\int_0^{2\pi} \sqrt{\frac{1 - \cos x}{2}} dx$ .

Answer : \_\_\_\_\_.

8. Let  $y = \frac{(x^2 + 1)(x + 3)^{\frac{1}{2}}}{x - 1}$ . Find  $\frac{dy}{dx} \Big|_{x=2}$ .

Answer : \_\_\_\_\_.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Suppose that  $F(x)$  is an antiderivative of

$$f(x) = \frac{\sin x}{x}, \quad x > 0.$$

Express  $\int_1^3 \frac{\sin 2x}{x} dx$  in terms of  $F$ .

2. (10 points) Investigate the convergence of

$$\int_1^{\infty} \frac{1 - e^{-x}}{x} dx.$$

3. (10 points) Evaluate the integral.

$$\int \frac{x^2 dx}{\sqrt{9 - x^2}}$$

4. (10 points) Evaluate the integrals.

a.  $\int \cos^3 \theta \sin 2\theta d\theta$

b.  $\int \sec^4 \theta d\theta$

5. (10 points) Show that

$$\int_a^b \left( \int_x^b f(t) dt \right) dx = \int_a^b (x - a)f(x) dx,$$

if  $f$  is continuous on  $[a, b]$ .

6. (10 points) Evaluate the integral.

$$\int_0^{\infty} \frac{dx}{(x + 1)(x^2 + 1)}$$

(試題結束)