

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Let $A(t)$ represent a quantity growing exponentially, then $A'(t)/A(t)$ is constant.
2. $\frac{d}{dx}b^x = xb^{x-1}$, where $b > 0$, $b \neq 1$.
3. The relative growth rate of P is the derivative of $\ln(P(t))$.
4. If f is not integrable on $[a, b]$, then f can't be a continuous function on $[a, b]$.
5. If f' is in Figure below, then $f(1) \approx f(3)$.
6. If $F'(t)$ is constant, then $\int_0^5 F'(t)dt = 0$.
7. If $\int_0^2 g(t)dt = 5$ then $\int_0^2 g(2-t)dt = -5$.
8. Indefinite integral is an antiderivative.
9. $f(x) = 1 - (1-x)^2$ is a Lorenz curve.
10. If $f(x) = f(-x)$, then $\int_{-2}^2 f(x) dx = 0$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the limit $\lim_{x \rightarrow 0} \frac{\sqrt{3x^2 + 1} - 1}{x^2}$.

Answer: _____.

2. Find the absolute extrema of $f(x) = x - 4\sqrt{x}$ on the interval $[0, 9]$.

Answer: _____.

3. A line goes through the origin and a point on the curve $y = x^2 e^{-3x}$, for $x \geq 0$.

Find the maximum slope of such a line.

Answer: _____.

4. Find the area of the region bounded by the graphs of the functions $f(x) = \frac{1}{x}$, $g(x) = -e^x$, $x = \frac{1}{2}$, $x = 1$.

Answer: _____.

5. Find the effective rate corresponding to the given nominal rate.

(a) 20%/year compounded semiannually (twice a year).

(b) 10%/year compounded continuously.

Write you answer in a pair: (the answer of (a), the answer of (b)).

If you fail to do so, you WILL suffer from NO POINTS.

Answer: (a) , (b) .

6. Compute the average value of $f(x) = 6x^2 - 17 - e^x$ over $[-1, 2]$. **If your answer has a term e^a where a is a number, please just leave that there.**

Answer: _____.

7. Find the coefficient of inequality (or Gini Index) of the Lorenz curve $y = f(x) = x^2$. Answer: _____.

8. Suppose that we know the following facts: (1) $\int_1^4 f(x)dx = 2$, (2) $\int_4^6 f(x)dx = -1$, (3) $\int_5^6 f(x)dx = 2$. Find $\int_1^5 4f(x) + e^{-x} dx$.

Answer: _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Neilsen Cookie Company sells its assorted butter cookies in containers that have a net content of 1 lb. The estimated demand for the cookies is 1,000,000 1-lb containers per year. The setup cost for each production run is \$500, and the manufacturing cost is \$0.50 for each container of cookies. The cost of storing each container of cookies over the year is \$0.40. Assume uniformity of demand throughout the year and instantaneous production.
 - a. Find the total cost function.
 - b. How many containers of cookies should Neilsen produce per production run in order to minimize the total cost?
2. (10 points) Four months after discontinuing advertising on national television, a manufacturer notices that sales have dropped from 100,000 MP3 players per month to 80,000 MP3 players. If the sales follow an exponential pattern of decline, what will they be after another 4 months?
3. (10 points) The value of good wine increases with age. Thus, if you are a wine dealer, you have the problem of deciding whether to sell your wine now, at a price of $\$P$ a bottle, or to sell it later at a higher price. Suppose you know that the amount a wine-drinker is willing to pay for a bottle of this wine t years from now is $\$P(1 + 20\sqrt{t})$. Assuming continuous compounding and a prevailing interest rate of 5% per year, when is the best time to sell your wine? **Note: Round the answer to the 2nd decimal place.** ($\sqrt{40.0025} \approx 6.32475296$)

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4. (10 points) Find

$$\int \frac{xe^{2x^2}}{e^{2x^2} + 1} dx.$$

5. (10 points) Define $f(x) = x^3$ and

$$g(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ -\sqrt{-x} & \text{if } x < 0 \end{cases}$$

(a) Sketch the graphs of the functions f and g .

(b) Find the area completely enclosed by the graphs of the functions f and g .

6. (10 points) Consider the demand function

$$p = D(x) = -x^2 + 20, 0 \leq x \leq 2\sqrt{5}$$

and the supply function

$$p = S(x) = x^2 + 2x + 8, 0 \leq x \leq 2\sqrt{5}.$$

(a) Find the equilibrium price and quantity. **Please make sure you do those correctly since you need them for part (b) and (c).**

(b) Find consumers' surplus if the market price is set at the equilibrium price.

(c) Find producers' surplus if the market price is set at the equilibrium price.

(試題結束)