微積分聯合教學　會考四　　Mar 31, 2015

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\sum_{n=1}^{\infty} a_n x^n$ is convergent at $x=-2$, then it is convergent at $x=1$.

2. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ is covergent.

3. If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are divergent series, then $\sum_{n=1}^{\infty}(a_n+b_n)$ is also divergent.

4. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty}\left(\frac{1+\sin a_n}{2}\right)^n$ converges.

5. $\lim_{n\to\infty} \frac{(\frac{10}{11})^n}{(\frac{9}{10})^n+(\frac{11}{12})^n}$ does not exist.

6. Symmetric about the x-axis and symmetric about the origin together imply symmetric about the y-axis.

7. If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} a_n^2$ converges.

8. The graphs of $r=-\frac{1}{2}$ and $r=1+\cos\theta$ have no intersection.

9. $\sum_{n=1}^{\infty}(a_n-a_{n+1})$ and $\{a_n\}$ both converge or both diverge.

10. The graph of $r=\frac{4}{2\cos\theta-\sin\theta}$ is a straight line through $(x,y)=(0,4)$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. For what values of p is the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

convergent? Answer：___________.

2. A curve C is parametrized by $x = t$ and $y = 1 - \cos t$, $0 \leq t \leq 2\pi$. Let l denote the largest slope generated by tangent lines of C, find l.

Answer：___________.

3. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ and $S_k = \sum_{n=1}^{k} \frac{(-1)^n}{n}$, where k is a positive integer. Find the largest value among S_8, S_9 and S.

Answer：___________.

4. Calculate $\sum_{n=1}^{\infty} \cos \frac{(2n+1)\pi}{2n(n+1)} \sin \frac{\pi}{2n(n+1)}$. Answer：___________.

5. Find the length of one arch of the cycloid $x = a(t - \sin t)$ and $y = a(1 - \cos t)$.

Answer：___________.

6. Find the Taylor polynomial of order n generated by $f(x) = \frac{1}{x}$ at $x = 3$.

Answer：___________.

7. Find $\lim_{n \to \infty} \left(\frac{3n+1}{3n-1} \right)^n$. Answer：___________.

8. Let $f(x) = 3^x = \sum_{n=0}^{\infty} a_n (x-1)^n$. Find the value of a_{10}.

Answer：___________.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Use the binomial series and the fact that $\frac{d}{dx}\sin^{-1} x = (1 - x^2)^{-\frac{1}{2}}$, $|x| < 1$, to generate the first four terms of the Taylor series of $\sin^{-1} x$.

(下頁還有試題)

2. (*10* points) Consider the sequence

$$\sqrt{3},\quad \sqrt{3+\sqrt{3}},\quad \sqrt{3+\sqrt{3+\sqrt{3}}},\quad \sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3}}}},\ldots$$

a. Show that the sequence is bounded.

b. Explain why the sequence is convergent.

c. Find the limit of the sequence.

3. (*10* points)

$$\sum_{n=1}^{\infty}\frac{(3x+1)^{n+1}}{2n+2}$$

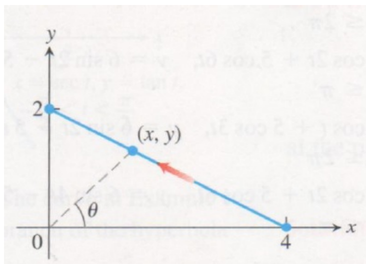
a. Find the series' radius and interval of convergence.

b. For what values of x does the series converge absolutely?

c. For what values of x does the series converge conditionally?

4. (*10* points) Find the area of the surface generated by revolving one arch of the cycloid $x = t - \sin t$, $y = 1 - \cos t$ about the x-axis.

5. (*10* points) Let L be a line segment joining points $(0, 2)$ and $(4, 0)$.



a. Find its Cartesian equation.

b. Find its Polar equation.

c. Find the parametrization for L using the angle θ in the accompanying figure as the parameter.

6. (*10* points) Determine which of the series converges absolutely, which converge conditionally and which diverge? Give reasons for your answers.

a. $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{\ln n}$

b. $\displaystyle\sum_{n=1}^{\infty}\frac{(-3)^{n+1}}{n+4^n}$

c. $\displaystyle\sum_{n=3}^{\infty}\frac{\ln n}{\ln\ln n}$

(試題結束)