

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The domain of the function $f(x, y)$ is $\{(x, y) \in R | x \neq y\}$ where $f(x, y) = \frac{1}{x^4 + y^4}$.

2. For any function $f(x, y)$, if the second derivatives $f_{xx}(x, y)$ and $f_{yy}(x, y)$ both exist, then $f_{xx}(x, y) = f_{yy}(x, y)$.

3. If f is an integrable odd function, then $\int_{-t}^t f(x)dx = 0$.

4. It is no way to find the volume of a solid with irregular shape even only approximate.

5. $\int e^x f(x)g'(x) dx = e^x f(x)g(x) - \int e^x f(x)g(x) dx - \int e^x f'(x)g(x) dx$.

6. Given $f(x) = ax^3 + bx + c$, $a \neq 0$,

$$\int_1^5 f(x) dx = \frac{1}{3} [f(1) + 4f(2) + 2f(3) + 4f(4) + f(5)].$$

7. $\int_{-\infty}^{\infty} x dx = \lim_{a \rightarrow \infty} \int_{-a}^a x dx = \lim_{a \rightarrow \infty} \left(\frac{1}{2} x^2 \Big|_{-a}^a \right) = 0$

8. Let $f(x, y) = x^2 - y^2$. Then $(0, 0)$ is the sole critical point of f but f has neither a relative maximum nor a relative minimum at $(0, 0)$.

(下頁還有試題)

9. Given 4 data points (1, 4), (2, 6), (3, 8), and (4, 11), the least-squares line passes through (\bar{x}, \bar{y}) , where $\bar{x} = \frac{1}{4}(1 + 2 + 3 + 4) = \frac{5}{2}$ and $\bar{y} = \frac{1}{4}(4 + 6 + 8 + 11) = \frac{29}{4}$.
10. Let $f(x, y) = 2x^2 + y^2$. The constrained minimum of f subject to $x + y - 1 = 0$ is larger than the unconstrained minimum.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $f(x, y) = e^{5x} + xy + \ln\left(\frac{x}{y}\right)$, then $f_{xy}(x, y) = \underline{\hspace{2cm}}$.
Answer: $\underline{\hspace{2cm}}$.
2. The integral of the form $\int_a^\infty e^{-px} dx$ is convergent if $p \underline{\hspace{2cm}}$.
Answer: $\underline{\hspace{2cm}}$.
3. The determinant D used in the second derivative test, which is used to determine the nature of a critical point of a function $f(x, y)$, is defined by $\underline{\hspace{2cm}}$.
Answer: $\underline{\hspace{2cm}}$.
4. Find the area of the region under the graph of $f(x) = xe^{-x}$ from $x = 0$ to $x = 2$. Answer: $\underline{\hspace{2cm}}$.
5. Use the Trapezoidal rule with $n = 3$ to approximate $\int_1^2 \frac{1}{x} dx$. Write your answer as a proper fraction. Answer: $\underline{\hspace{2cm}}$.
6. Find the volume of the solid obtained by revolving the region bounded by the curves $y = \sqrt{x}$ and $y = x^3$ about the y -axis. Answer: $\underline{\hspace{2cm}}$.
7. The demand equations for two commodities are given by $x = 10000 - 10p - e^{0.5q}$ and $y = 50000 - 4000q - 10p$, respectively. Determine whether these two products are substitute, complementary, or neither. Answer: $\underline{\hspace{2cm}}$.
8. Find the minimum of the function $f(x, y, z) = xy + 2yz + 2xz$ subject to the constraint $xyz = 108$. Answer: $\underline{\hspace{2cm}}$.

(下頁還有試題)

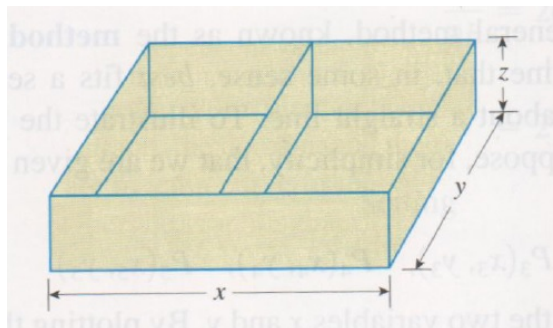
計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) In a study, the Lorenz curve for the distribution of income of college professors is described by the function $f(x) = \frac{1}{3}x\sqrt{1+8x}$. Compute the coefficient of inequality (Gini Index) of the Lorenz curve.

$$\left(\text{Hint: } \int u(a+bu)^{0.5} du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{1.5} + \text{constant.} \right)$$

2. (10 points) Mr. Who wishes to establish a fund to provide a Business School with an annual research grant of \$1,000,000 beginning next year. If the fund will earn an interest rate of 10%/year compounded continuously, find the amount of the endowment he is required to make now.

3. (10 points) An open box having a volume of 48 in^3 . is to be constructed. If the box is to include a partition that is parallel to a side of the box, as shown in the figure, and the amount of material used is to be minimal, what should be the dimensions of the box?



4. (10 points) Evaluate the following integrals.

a. $\int_{1/2}^1 x^2 \ln 2x dx$

b. $\int_0^1 \frac{1}{1+e^{-x}} dx$

5. (10 points) The demand function for Apex women's boots is $p = \frac{250}{\sqrt{16+x^2}}$, where p is the unit price in dollars and x is the quantity demanded daily, in units of a hundred. Find the consumers' surplus if the price is set at \$50/pair.

$$\left(\text{Hint: } \int \frac{du}{\sqrt{a^2+u^2}} = \ln |u + \sqrt{a^2+u^2}| + C. \right)$$

(下頁還有試題)

6. (10 points) Ross-Simons Company has a monthly advertising budget of \$60,000. Their marketing department estimates that if they spend x dollars on newspaper advertising and y dollars on television advertising, then the monthly sales will be given by $f(x, y) = 90x^{1/4}y^{3/4}$ dollars. Determine how much money Ross-Simons should spend on newspaper ads and on television ads each month to maximize its monthly sales.

(試題結束)