微積分聯合教學　會考五　**May 12, 2015**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f(x,y)$ is a differentiable function and the point (x_0, y_0) in the domain of f, then $\nabla f(x_0, y_0)$ is normal to the level curve through (x_0, y_0).

2. If $|\mathbf{r}(t)| = 1$ for all t, then $|\mathbf{r}'(t)|$ is a constant.

3. If f has a local minimum at (a,b), then $\nabla f(a,b) = \mathbf{0}$.

4. There exists a function f with continuous second-order partial derivatives such that $f_x(x,y) = y$ and $f_y(x,y) = x$.

5. If the partial derivatives f_x and f_y of a function $f(x,y)$ are continuous throughout an open region R, then $f(x,y)$ is continuous on R.

6. Suppose $\mathbf{a}, \mathbf{b} \neq \mathbf{0}$. If $\mathbf{a} = -\mathbf{b}$, then $\text{proj}_{\mathbf{a}}\mathbf{b} = -\,\text{proj}_{\mathbf{b}}\mathbf{a}$

7. $[\mathbf{a} \times (\mathbf{b} \times \mathbf{c})] \cdot (\mathbf{b} \times \mathbf{c}) = 0$

8. Let $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$ be a smooth vector function. If $|\mathbf{r}(t)| = 1$ for all t, then $|\mathbf{r}(t) \times \mathbf{r}'(t)| = |\mathbf{r}'(t)|$.

9. Let $w = xy + \dfrac{e^y}{y^2+1}$. Then $\dfrac{\partial}{\partial x}\left(\dfrac{\partial w}{\partial y}\right) = 1$.

10. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2} = 0$.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find $\dfrac{\partial z}{\partial y}(0,1,2)$ if $x^3+y^3+z^3+6xyz=9$.

Answer：___________.

2. Find the linearization $L(x,y)$ of the function $f(x,y)=xe^y+\cos xy$ at $(2,0)$.

Answer：___________.

3. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-\cos(x+y^2)}{x+y^2}$.

Answer：___________.

4. Find the length of the curve

$$\mathbf{r}(t)=\left(\sqrt{2}t\right)\mathbf{i}+\left(\sqrt{2}t\right)\mathbf{j}+(1-t^2)\mathbf{k}$$

from $\left(\sqrt{2},\sqrt{2},0\right)$ to $\left(2\sqrt{2},2\sqrt{2},-3\right)$. Answer：___________.

5. $\mathbf{r}(t)=(3t+1)\,\mathbf{i}+\sqrt{3}\,t\,\mathbf{j}+t^2\,\mathbf{k}$ is the position of a particle in $\mathbf{R}^3$ at time t. Find the angle between the velocity and acceleration vectors at time $t=1$.

Answer：___________.

6. Find the area of the triangle whose vertices are $A(-5,3), B(1,-2), C(6,-2)$.

Answer：___________.

7. Find the distance from the point $(1,1,5)$ to the line

$$L:\quad x=1+t,\quad y=3-t,\quad z=2t.$$

Answer：___________.

8. Express $\dfrac{\partial w}{\partial s}$ in term of r and s if

$$w=x+2y+z^2,\quad x=\frac{r}{s},\quad y=r^2+\ln s,\quad z=2r.$$

Answer：___________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Let

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , (x,y) = (0,0) \end{cases} .$$

Find the partial derivative $\dfrac{\partial f}{\partial x}$ at $(x,y)=(0,0)$ and at $(x,y)\neq(0,0)$.

2. (*10* points) Find an equation of the tangent plane and find parametric equations for the normal line to the surface $x^2+y^2+z^2=6xyz-3$ at the point $(-1,1,-1)$.

3. (*10* points) Find the curvature for the plane curve

$$\mathbf{r}(t) = t\mathbf{i} + (\ln\cos t)\mathbf{j}, \quad -\pi/2 < t < \pi/2.$$

4. (*10* points) Find the absolute maximum of

$$f(x,y) = 5-2x-2y+x^2+y^2$$

on the triangular region in the first quadrant bounded by the lines $x=0$, $y=0$, $y=4-x$.

5. (*10* points) Fine parametric equations for the line in which the planes $3x-6y-2z=3$ and $2x+y-2z=2$ intersect.

6. (*10* points)

a. Find the derivative of $f(x,y,z)=x^3-xy^2-z$ at $P_0(1,1,0)$ in the direction of $\mathbf{v}=2\,\mathbf{i}-3\,\mathbf{j}+6\,\mathbf{k}$.

b. In what direction does f increase most rapidly at P_0?

(試題結束)