管院微積分聯合教學　會考五　**May 12, 2015**

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。 (不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $y = f(x)$ is a solution of a differential equation, then $y = f(x) + C$ is also a solution for any $C \in \mathbf{R}$.

2. Assume that $y = f(x)$ is a solution to the equation $dy/dx = 2x - y$. Then there could be more than one value of x such that $f'(x) = 1$ and $f(x) = 5$.

3. There is a solution curve for the logistic differential equation $dP/dt = P(2-P)$ that goes through the points $(0,1)$ and $(1,3)$.

4. If $f(x,y)$ is a function of y only, then $\int_a^b \int_0^1 f\ dxdy = \int_a^b f\ dy$.

5. Let $f(x,y)$ be a joint probability density function for the random variables X and Y on $\mathbf{R}^2 = \{(x,y) | -\infty < x < \infty; -\infty < y < \infty\}$. Then $\int_a^b f(x,y)\ dx = P(a \le X \le b)$.

6. If two functions f and g have the same total differential at the point $(1,1)$, then $f = g$.

7. The differential equation $y' = M(x,y)N(x,y)$ is separable if both $y' = M(x,y)$ and $y' = N(x,y)$ are separable.

8. A random variable can take finite values only.

(下頁還有試題)

9. Suppose that X has the exponential density function f with parameter k. If k goes to ∞, then $E(X)$ tends to 0.

10. Let X be a discrete random variable with mean 0 and suppose that $c > 1$, then $Var(X) > Var(cX)$.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Use the total differential of $f(x, y) = \sqrt{x^2 + y^3}$ at the point $(1, 2)$ to estimate $f(1.04, 1.98)$.

Answer:___________.

2. Find the volume of the solid bounded above by $z = f(x, y) = e^{-x^2}$ and below by the plane region R which is bounded by $y = x$, $x = 1$ and $y = 0$.

Answer:___________.

3. According to the economist Vilfredo Pareto (1848-1923), the rate of decrease of the number of people y in a stable economy having an income of at least x dollars is directly proportional to the number of such people and inversely proportional to their income x. This is modeled by the differential equation $\frac{dy}{dx} = -k\frac{y}{x}$. Find the general solution of this differential equation.

Answer:___________.

4. A new drug is introduced through an advertising campaign to a population of 1 million potential customers. The rate at which the population hears about the drug is assumed to be proportional to the number of people who are not yet aware of the drug. By the end of 1 year, half of the population has heard of the drug. How many will have heard of it by the end of 2 years?

Answer:___________.

5. The function $f(x) = 12x^2$ is a probability density function over $[-a, a]$. What is a? Answer:___________.

(下頁還有試題)

6. In 2006, the scores for the Graduate Management Admission Test (GMAT) could be modeled by a normal probability density function with a mean of $\mu = 527$ and a standard deviation of $\sigma = 117$. If you select a person who took the GMAT in 2006, what is the probability that the person scored between 585.5 and 702.5? $[P(Z < 0.5) = 0.6915, P(Z < 1.5) = 0.9332]$

Answer:___________.

7. The population density of a certain city is described by the function $f(x,y) = e^{2|x|+3|y|}$. What is the average population inside the rectangular area $\{(x,y) : 0 \le x \le 2, 0 \le y \le 2\}$.

Answer:___________.

8. Let $f(x,y) = 2e^{-2x}\dfrac{1}{y^2}$ be the joint probability density function for the random variables X and Y on $D = \{(x,y) : 0 \le x < \infty, 1 \le y < \infty\}$. Find $P\{0 \le X \le 1, 2 \le Y < \infty\}$.

Answer:___________.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Evaluate $\displaystyle\int_0^6 \int_{x/3}^2 x\sqrt{y^3+1}\, dydx.$

2. (*10* points) Find the particular solution of the differential equation

$$x\frac{dy}{dx} = \frac{\ln x}{7}$$

with the condition $y = -2$ when $x = 1$.

3. (*10* points) Use Euler's method with $n = 4$ to obtain an approximation of the solution of the initial value problem $y' = 0.5x + (3 - y)$, $y(0) = 1$ when $x = 2$.

(下頁還有試題)

4. (*10* points) A point is chosen at random from the region S in the $xy-$plane containing all points (x, y) such that $-1 \leq x \leq 1$, $-2 \leq y \leq 2$and $x - y \geq 0$ ("at random" means that the probability density function is constant on S). If T is a subset of S with area a, find the probability that a point (x, y) is in T.

5. (*10* points) A simple mathematical model in epidemiology for the spread of a disease assumes that the rate at which the disease spreads is jointly proportional to the number of infected people and the number of uninfected people. Suppose that there are a total of N people in the population, of whom N_0 are infected initially. Suppose that $x(t)$ denotes the number of infected people after t weeks. Solve $x(t)$.

Hint: you may need the following formula $\frac{1}{ab} = \frac{1}{a+b}\left(\frac{1}{a} + \frac{1}{b}\right)$.

6. (*10* points) Suppose that a random variable X has the following density function: $f(x) = 2x$, over the interval $[0, 1]$.

(a) Verify that $f(x)$ is a probability density function. (2 points)

(b) Compute $E(X)$. (4 points)

(c) Find $Var(X)$. (4 points)

(試題結束)