微積分聯合教學　　會考六　　Jun 23, 2015

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*30* points)，請答 **T** (True) 或 **F** (False)。每題 3 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\sum_{n=1}^{\infty} a_n x^n$ is convergent at $x = -2$, then it is convergent at $x = 1$.

2. If the partial derivatives f_x and f_y of a function $f(x, y)$ are continuous throughout an open region R, then $f(x, y)$ is continuous on R.

3. The vector field $\mathbf{F}(x, y) = \dfrac{-y}{x^2 + y^2}\mathbf{i} + \dfrac{x}{x^2 + y^2}\mathbf{j}$ is conservative in its natural domain.

4. $2xydx + (x^2 - z^2)dy - 2yzdz$ is exact.

5. The line integral of the constant function $f(x, y, z) = 2$ over a smooth curve C of length 5 is 10.

6. If $\mathbf{F}$ is a gradient vector field, then the line integral of $\mathbf{F}$ along every curve is zero.

7. If D is the disk given by $x^2 + y^2 \le 4$, then $\displaystyle\int\int_D \sqrt{4 - x^2 - y^2}\, dA = \frac{16}{3}\pi$.

8. If R is a region in the plane bounded by a piecewise smooth, simple colsed curve C, then Area of $R = -\oint_C ydx$.

9. The area of the fan-shaped region between $\theta = \alpha$, $\theta = \beta$ and the curve $r = f(\theta)$, $f(\theta) \ge 0$, $\alpha \le \theta \le \beta$, is $\displaystyle\int_\alpha^\beta \int_0^{f(\theta)} drd\theta$.

(下頁還有試題)

10. The value of the line integral along a path joining two point can change if you change the path between them.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find an equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 6xyz - 3$ at the point $(-1, 1, -1)$. Answer：____________.

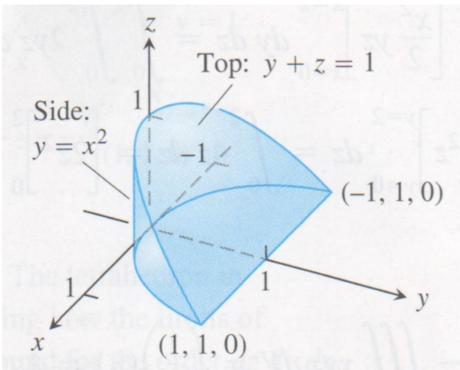
2. Find the work done by the force $\mathbf{F} = (y - x)\mathbf{i} - xy\mathbf{j}$ over the straight line from $(1, 1)$ to $(3, 2)$. Answer：____________.

3. Convert $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{1} (x^2 + y^2)\ dzdydx$ to an equivalent integral in spherical coordinates. Answer：____________.

4. Find the Jacobian $\partial(x, y)/\partial(u, v)$ of the transformation $u = x + 2y$, $v = x - y$.

Answer：____________.

5. Rewrite the integral $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dzdydx$ as an equivalent iterated integral in the order $dydzdx$.



Answer：____________.

6. $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} x^2 + y^2\ dydx$. Answer：____________.

7. Find the average value of $f(x, y) = \sin(x + y)$ over the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq \pi$.

Answer：____________.

8. Evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz$ over any path from $(1, 1, 1)$ to $(2, 3, -1)$. Answer：____________.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*50* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the length of one arch of the cycloid $x = a(t - \sin t)$ and $y = a(1 - \cos t)$.

2. (*10* points) Sketch the region of integration, reverse the order of integration, and evaluate the integral.

$$\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} \, dydx.$$

3. (*10* points) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

4. (*10* points) Show that $\mathbf{F} = (e^x \cos y + yz)\mathbf{i} + (xz - e^x \sin y)\mathbf{j} + (xy + z)\mathbf{k}$ is conservative over its natural domain and find a potential function for it.

5. (*10* points) Evaluate

$$\int_0^4 \int_{x=y/2}^{x=(y/2)+1} \frac{2x - y}{2} \, dxdy$$

by applying the transformation $u = \dfrac{2x - y}{2}$, $v = \dfrac{y}{2}$ and integrating over an appropriate region in the uv-plane.

6. (*10* points) Evaluate the line integral $\oint_C xydy - y^2 dx$, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

(試題結束)