

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f is an integrable odd function, then $\int_{-1}^1 f(x)dx = 0$.
2. Suppose that X has the exponential density function f with parameter k . If k goes to ∞ , then $E(X)$ tends to 0.
3. $\int_{-3\pi/2}^{3\pi/2} \cos(x)dx = 0$.
4. If the internal rate of return of stock A is less than the internal rate of return of stock B, then invest stock B is a better decision.
5. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (x-1)^n$ is the Taylor series of $\ln x$ at $x = 0$.
6. The integral test can be applied to any kind of series.
7. If $|r| < 1$, then $\sum_{n=1}^{\infty} \frac{1}{r^n}$ diverges.
8. $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{2n^2+1}} = \frac{1}{\sqrt{2}} < 1$, so the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{2n^2+1}}$ converges.
9. Let $\sum a_n$, $\sum b_n$, and $\sum c_n$ be series with positive terms. If $\sum a_n$ is divergent and $b_n + c_n \geq a_n$ for all n , then both $\sum b_n$ and $\sum c_n$ are divergent.
10. $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2} = -\frac{1}{2}$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the particular solution of the differential equation $x \frac{dy}{dx} = \frac{\ln x}{7}$ with the condition $y = -2$ when $x = 1$.

Answer: _____.

2. Evaluate $\lim_{x \rightarrow 0} x \cot(x)$.

Answer: _____.

3. Find the derivative of the function $f(x) = x^2 \sin\left(\frac{1}{x}\right)$.

Answer: _____.

4. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} (2n)!(x-1)^n$.

Answer: _____.

5. Find the sum of the series $\sum_{n=2}^{\infty} \left[\frac{1}{3^n} - \frac{1}{n(n+1)} \right]$.

Answer: _____.

6. Find all positive values of p for which the series $\sum_{n=2}^{\infty} \frac{1}{n^2(\ln n)^p}$ is convergent.

Answer: _____.

7. Given $\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots$, $-1 < x < 1$, find the Taylor series and its interval of convergence of $f(x) = \frac{1}{1+x}$ at $x = 1$.

Answer: _____.

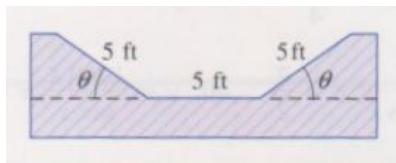
8. Find the area of the region bounded by the graph of $f(x) = x \cos(x)$, the x -axis, and the lines $x = -\pi$ and $x = \pi$.

Answer: _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the minimum of the function $f(x, y, z) = xy + 2yz + 2xz$ subject to the constraint $xyz = 108$.
2. (10 points) Use the second Taylor polynomial of $f(x) = e^{-2x}$ at $x = 0$ to approximate $e^{-0.2}$, and find a bound for the error in the approximation.
3. (10 points) The cross section of a drain is a trapezoid, as shown in the accompanying figure. The sides and the bottom of the trapezoid are each 5 ft long. Determine the angle θ such that the drain will have a maximal cross-sectional area.



4. (10 points) Of the microprocessors manufactured by a microelectronics firm for use in regulating fuel consumption in automobiles, 1.5% are defective. Let a_n denote the probability of getting at least one defective microprocessor in a random sample of n microprocessors.
 - a. Find the formula for a_n .
 - b. Evaluate $\lim_{n \rightarrow \infty} a_n$.
5. (10 points) Suppose that the average wage earner save 9% of her take-home pay and spends the other 91%. Estimate the impact that a proposed \$30 billion tax cut will have on the economy over the long run due to the additional spending generated.
6. (10 points) Suppose that the rate of air flow into and out a person's lungs during respiration is $R(t) = 0.6 \sin \frac{\pi t}{2}$ liters per second, where t is the time in seconds. Find an expression for the volume of air V in the person's lung at any time t . Assume the $V(0) = 0$.

(試題結束)