

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Suppose $\lim_{x \rightarrow 2} \frac{f(x) - 5}{x - 2} = 3$, then $\lim_{x \rightarrow 2} f(x) = 5$.
2. If $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 4$, then $\lim_{x \rightarrow 0} \frac{f(x)}{x} = \lim_{x \rightarrow 0} f(x)$.
3. There exists a function $y = f(x)$ which has three vertical asymptotes.
4. The function $y = x^2 \sin \frac{1}{x} + 1$ has a horizontal asymptote.
5. $f(x) = \frac{x}{x+1}$ is symmetric with respect to the point $(-1, 1)$.
6. $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x} = 1$.
7. If $f'(a)$ does not exist, then it is still possible that f is continuous at a .
8. $x^{100} - 10x + 1$ has at least one root in $[0, 2]$.
9. If $f(x)$ is an even function and $f'(c) = 1$, then $f'(-c) = 1$.
10. If $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f'(0) = 0$.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. If there is a number k that makes

$$f(x) = \begin{cases} \frac{\sin^2 x}{4x^2}, & x > 0 \\ k + \cos x, & x \leq 0 \end{cases}$$

continuous at $x = 0$, then find k .

Answer : _____.

2. Evaluate $\lim_{x \rightarrow 0} \frac{\tan(2x)}{3x}$.

Answer : _____.

3. Find an equation of the normal line to the parabola $y = x^2 - 2x + 3$ that is parallel to $y = 4x + 5$.

Answer : _____.

4. Find the linearization of

$$y = \cos(x^2 + x) + \frac{1}{2x + 1}$$

at $x = 0$. Answer : _____.

5. Evaluate $\lim_{x \rightarrow 0} \frac{8x}{x - 3 \sin x}$.

Answer : _____.

6. Let $f(x) = \frac{1}{\sqrt{x-1}}$. Find the domain of the composite function $f \circ f$.

Answer : _____.

7. Find $\lim_{x \rightarrow 0^+} \left[\left(\sin \frac{1}{x} \right) (\cos x) \right]$.

Answer : _____.

8. If $\lim_{x \rightarrow 0^+} f(x) = A$, $\lim_{x \rightarrow 0^-} f(x) = B$. Find $\lim_{x \rightarrow 0^-} f(x^4 - x^2)$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the tangent line to the curve $x^2 \cos^2 y - \sin y = 0$ at $(0, \pi)$.
2. (10 points) A rock is thrown into a still pond and causes a circular ripple (漣漪). If the radius of the ripple is increasing at 2 feet per second. How fast is the area changing when the radius is 10 feet?
3. (10 points) Find the derivative

$$y = \tan \left(2 \sec \sqrt{x^2 + \frac{1}{3x}} \right).$$

Please no need to simplify your answer.

4. (10 points) Let $f(x) = |x^2 - 3x|$.
 - a. At which points is $f(x)$ continuous?
 - b. At which points is $f(x)$ differentiable?Please explain why.

5. (10 points) Use $\epsilon - \delta$ definition of limit to prove that

$$\lim_{x \rightarrow 0} \sqrt{4 - x} = 2.$$

6. (10 points) Find all asymptotes of the graph of $y = \frac{2x^2}{1 - x}$.

(試題結束)