

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Suppose the slope of a line L is $-\frac{1}{3}$ and P is a given point on L . If Q is the point on L lying 6 units to the right of P , then Q is situated 2 units below P .
2. A vertical line intersects the graph of $y = f(x)$ in at most one point.
3. Suppose the function f is continuous on the interval $[a, b]$. If $f(a)$ and $f(b)$ have the same sign, then f has no zero in $[a, b]$.
4. If $f(x)$ is continuous at $x = a$ and g is differentiable at $x = a$, then
$$\lim_{x \rightarrow a} f(x)g(x) = f(a)g(a).$$
5. If $f(x)$ is differentiable, then
$$\frac{d}{dx} \left[\frac{1}{f\left(\frac{1}{x}\right)} \right] = \frac{f'\left(\frac{1}{x}\right)}{x^2 \left[f\left(\frac{1}{x}\right) \right]^2}.$$
6. If the demand is inelastic at p , then a slight increase in the unit price will cause the revenue to increase.
7. For any given functions $f(x)$ and $g(x)$, we always have $f(g(x)) = g(f(x))$.
8. The functions $f(x) = x + 3$ and $g(x) = (x^2 - 9)/(x - 3)$ have the same domain.
9. There is a function f for which f is not continuous at 3 but f' exists at 3.
10.
$$\frac{d}{dx} [f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x).$$

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Suppose the fatality rate (deaths per 100 million miles traveled) of motorcyclists is given by $g(x)$, where x is the percentage of motorcyclists who wear helmets. Next, suppose the percent of motorcyclists who wear helmets at time t (t measured in years) is $f(t)$, with $t = 0$ corresponding to 2000. If $f(6) = 0.51$ and $g(0.51) = 42$, find the fatality rate in the year 2006. Answer: _____.

2. A company has a fixed cost of \$30,000 and a production cost of \$6 for each unit it manufactures. A unit sells for \$10. What is the profit function for manufacturing and selling x units? Answer: _____.

3. Evaluate $\lim_{x \rightarrow -\infty} \frac{|2x|}{x}$.
Answer: _____.

4. The concentration of a certain drug in a patient's bloodstream t hr after injection is given by

$$C(t) = \frac{0.2t}{t^2 + 1}.$$

How fast is the concentration changing 2 hr after the injection?

Answer: _____.

5. The demand function for a product is given by

$$d = x^2 - 12x + 27 \quad (0 \leq x \leq 3)$$

and the corresponding supply function is given by

$$s = x^2 + 15 \quad 0 \leq x \leq 3$$

where d and s are in dollars and x is measured in units of a **thousand**. Find the equilibrium quantity q_0 and equilibrium price p_0 and write you answers in a pair (q_0, p_0) . **Failing to do so may lose ALL points.**

Answer: _____.

(下頁還有試題)

6. Suppose that $\lim_{x \rightarrow 3} f(x) = 2$, $\lim_{x \rightarrow 2} g(x) = -1$, $\lim_{x \rightarrow 3} h(x) = 1$ and $g'(3) = -1$, find

$$\lim_{x \rightarrow 3} \frac{g(f(x)) + x^2}{h^2(x)}.$$

Answer: _____.

7. Consider the limit

$$\lim_{x \rightarrow \infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0},$$

where a_0, a_1, \dots, a_n and b_0, b_1, \dots, b_m are all real numbers and $a_n \neq 0, b_m \neq 0$.

When the limit exists? Answer: _____.

8. Suppose that we have

$$f'(a) = \lim_{x \rightarrow 3} \frac{\sqrt{x^2 + 6} - \sqrt{15}}{x - 3}.$$

Find f and a . **When you fill in your answer sheet, write f first and then a for getting full points.** Answer: _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the indicated limit.

a. $\lim_{x \rightarrow 3} \frac{x\sqrt{x^2 + 7}}{2x - \sqrt{2x + 3}}$

b. $\lim_{x \rightarrow 4^-} \frac{x - 4}{\sqrt{x} - 2}$

2. (10 points) The demand equation for a certain make of exercise bicycle sold exclusively through cable television is $p = \sqrt{9 - 0.02x}$, $0 \leq x \leq 450$, where p is the unit price in hundreds of dollars and x is the quantity demanded per week.

a. Compute the elasticity of demand.

b. Determine the range of prices corresponding to inelastic, unitary, and elastic demand.

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3. (10 points) Find the derivative of each function.

a. $f(x) = (x^2 - 5x + 3) \left(\frac{x-2}{x} \right)$

b. $h(t) = \left(\frac{t^2 + 1}{\sqrt{t}} \right)^{3/2}$

4. (10 points) Let $f(x) = \sqrt{x+1}$, find $f'(x)$ **BY DEFINITION**.

5. (10 points) Suppose that a high speed train moves along a straight line and its position after t seconds is given by a function $P(t) = 3t^2 + 2t$ (in kilometers) ($0 \leq t \leq 60$).

a. Calculate the average velocity of the train over the time interval $[30, 31]$.

b. Calculate the velocity of the train when $t = 30$.

c. Compare the results obtained in part (a) with that obtained in part (b).

6. (10 points) Let

$$f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ 0, & x < 0 \end{cases},$$

a. Indicate which points the function is continuous at.

b. What is the domain of $f'(x)$?

It is necessary to show all of your work and find a reasonable way to support your answers. If not, you may lose ALL points!!!!.

(試題結束)