

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $f'(c) = 0$, then f has a local maximum or minimum value at c .
2. If $f''(c) = 0$, then $(c, f(c))$ is a point of inflection of the curve $y = f(x)$.
3. There exists a function having a point of inflection where neither the first nor the second derivative exists.
4. Suppose that f has a positive derivative for all values of x and that $f(1) = 0$.
Let $g(x) = \int_0^x f(t)dt$. Then g has a local maximum at $x = 1$.
5. $\int_{-\pi}^{\pi} x^2 \sin(x^3)dx = 0$.
6. $\sum_{k=-1}^4 (2k+1) = \sum_{k=0}^5 (2k-1)$.
7. If $f(x)$ is continuous on $[0, 1]$, then $\int_0^1 f(x) dx = \int_0^1 f(1-x) dx$.
8. $\int x \sin x dx = \frac{x^2}{2} \sin x + C = -x \cos x + \sin x + C$.
9. The region bounded by the curve $y = \sqrt{x}$, the x -axis, and the line $x = 4$ is revolved about the x -axis to generate a solid. Let V be the volume of that solid.

$$V = \int_0^4 \pi(\sqrt{x})^2 dx = \int_0^2 2\pi y(4-y^2) dy.$$

(下頁還有試題)

10. Let $x \geq 0$ and $y = \sqrt{x}$. $\int_0^{a^2} \sqrt{1 + \frac{1}{4x}} dx = \int_0^a \sqrt{1 + 4y^2} dy$, for any positive number a .

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Use integral to evaluate

$$\lim_{n \rightarrow \infty} \frac{1 + 2^5 + 3^5 + \dots + n^5}{n^6}.$$

Answer : _____.

2. Find the average value of $f(x) = 2 - |x|$ on $[-1, 1]$.

Answer : _____.

3. Find the area of the region enclosed by the curves $x + y^2 = 1$ and $2x + y^2 = 0$.

Answer : _____.

4. Find the length of the graph of $f(x) = \frac{x^3}{12} + \frac{1}{x}$ from $x = 1$ to $x = 2$.

Answer : _____.

5. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 3$, about the x -axis.

Answer : _____.

6. Suppose that $F(x)$ is an antiderivative of $f(x) = \frac{\sin x}{x}$, $x > 0$. Express

$$\int_1^3 \frac{\sin 2x}{x} dx \text{ in terms of } F.$$

Answer : _____.

7. Evaluate the definite integral $\int_0^\pi \sin^2 x \cdot \cos^2 x dx$.

Answer : _____.

8. Let a be the absolute maximum value of $f(x) = x^2 - 1$ on $-1 \leq x \leq 2$, and b be the absolute minimum value. Find $a + b$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points)

- a. Please describe the Mean Value Theorem.
- b. Prove the inequality

$$|\cos x - \cos y| \leq |x - y|$$

for all x and y .

2. (10 points) A wire 10 m long is cut into two pieces. One piece is bent into an equilateral triangle and the other is bent into a circle. If the sum of the areas enclosed by each part is a minimum, what is the length of each part?

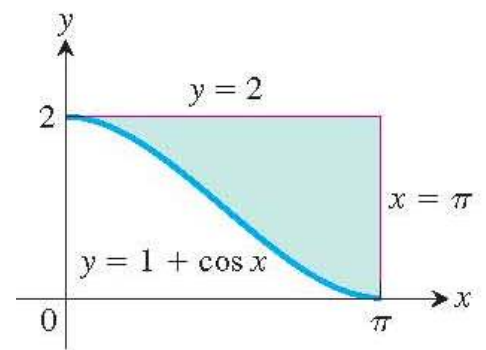
3. (10 points)

- a. Find $\int x^3 \sqrt{x^2 + 1} dx$.
- b. Find $\frac{d}{dx} \int_0^{\sqrt{x}} |\cos t| dt, x > 0$.

4. (10 points) The region in the first quadrant (象限) bounded by the curve $y = x^2$, the x -axis and the line $x = 2$ is revolved about the line $x = -1$ to generate a solid. Compute the volume of the solid.

5. (10 points) Let $f(x) = \frac{x^2 - 3}{x - 2}, x \neq 2$. Find **a.** critical numbers, **b.** intervals of increasing/decreasing, **c.** local maximum/minimum values, **d.** intervals of concavity, **e.** inflection points, **f.** oblique, horizontal and vertical asymptotes. **g.** Then sketch the graph of $f(x)$.

6. (10 points) Let $f(x) = 1 + \cos x$. Find **a.** the area of the shaded region, and **b.** the average value of f on $[0, \pi]$.



(試題結束)