管院微積分聯合教學　　會考二　　　　　　　**Dec 1, 2015**

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. To find critical numbers, it is sufficient to find all x in the domain with $f'(x) = 0$.

2. Let $f(x) = \ln x$ and $g(x) = e^x$. Then $f(g(x))$ and $g(f(x))$ have the same domain.

3. If f is continuous on (a, b), then f has the absolute maximum value.

4. If f is continuous on $[a, b]$ and $f'(c) = 0$ for some $c \in [a, b]$, then f has an absolute extrema at c.

5. If f is continuous on $[1, 5]$ and $f'(3)$ does **NOT** exist, then f does **NOT** have relative extreme value at 3.

6. $\left(8x^{\frac{1}{18}}\right)^{\frac{1}{3}} = 2x^6$.

7. $\ln \left[\dfrac{x^3}{\sqrt{x}(2+x)^3}\right] = \dfrac{5}{2}\ln x - 3\ln(2+x)$.

8. If the graph of f is concave upward on (a, c) and concave downward on (c, b), where $a < c < b$, then f has an inflection point at $(c, F(c))$.

9. The function $f(x) = \dfrac{1}{\ln(x-1)}$ is continuous on $[1, \infty)$.

10. If f is not continuous on the closed interval $[a, b]$, then f cannot have an absoluted minimum value.

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find **ALL** inflection points of the graph $y = f(x) = x^3 + 6x^2 + 9x + 7$.

   Answer:_____.

2. Suppose that the position of a maglev satisfies $f(t) = t^3 - \sqrt{t} + t^2$, $t \geq 0$. What is the maglev's acceleration at time 1.

   Answer:_____.

3. Approximate the value of $126^{1/3}$ using differentials.

   Answer:_____.

4. Find **ALL** vertical and horizontal asymptotes of the following function:

$$f(x) = \frac{2x^3}{x^3 - 27.}$$

   Answer:_____.

5. Determine where the graph of the function

$$f(x) = \log \frac{e^{2x^3 + 12x}}{e^{9x^2 + 7}}$$

   is increasing.

   Answer:_____.

6. Let $f(x) = \left(\frac{7}{4}x^3 + 5x - 8 \right)^{7.5}$. Find $f''(x)$.

   Answer:_____.

7. Find the second derivative $\frac{d^2y}{dx^2}$ of the function defined implicitly by the equation $y^2 - 13xy = 52$.

   Answer:_____.

8. Let $f(x) = \frac{9}{5x - 4}$ if $x \in (2, \infty)$. Find the critical number of $f(x)$.

   Answer:_____.

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計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Sketch the graph of the function $f(x) = x^4 - 2x^2$. Indicates all the points that matters. **In order to get points, you should use derivatives instead of plotting points. Show all your work but don't put nonsense, please**.

2. (*10* points) The concentration (in milligrams per cubic centimeter) of a certain drug in a patient's bloodstream t hours after injection is given by

$$C(t) = \frac{t}{t^2 + 2}, \quad t \geq 0.$$

(a) When does the concentration in bloodstream achieve the maximum?

(b) What happen if time t goes to ∞? Interpret your result.

Please write everything in detail.

3. (*10* points) The volume of a spherical cancerous tumor is given by

$$V(r) = \frac{4}{3}\pi r^3, \quad r > 0.$$

If the radius of a tumor is estimated at 1.5 cm, with a maximum error in measurement of 0.01 cm, determine the error that might occur when the volume of the tumor is calculated.

4. (*10* points) Phillip, the proprietor of a vineyard, estimates that the first $10,000$ bottles of wine produced this season will fetch a profit of \$2/bottle. However, the profit from **each bottle beyond** $10,000$ drops by \$0.0004 for **each additional bottle** sold. Assuming at least $10,000$ bottles of wine are produced and sold, what is the maximum profit?

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5. (*10* points) During daylight hours, some birds fly more slowly over water than over land because some of their energy is expended in overcoming the downdrafts of air over open bodies of water. Suppose a bird that flies at a constant speed of 4 mph over water and 6 mph over land starts its journey at the point E on an island and ends at its nest N on the shore of the mainland, as shown in the accompanying figure. Find the location of the point P that allows the bird to complete its journey in the minimum time (solve for x).
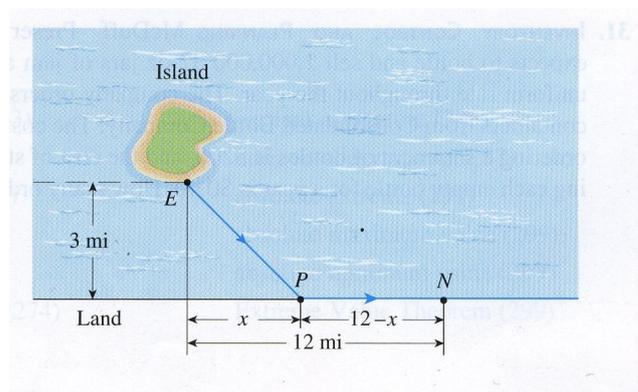
6. (*10* points) A car leaves an intersection traveling west. Its position 4 sec later is 20 ft from the intersection. At the same time, another car leaves the same intersection heading north so that its position t sec later is $t^2 + 2t$ ft from the intersection. If the speed of the first car 4 sec after leaving the intersection is 9 ft/sec, find the rate at which the distance between the two cars is changing at that instant of time.

(試題結束)

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