微積分聯合教學　會考三　**Jan 12, 2016**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. There exists a function $y = f(x)$ which has three vertical asymptotes.

2. Let $x \geq 0$ and $y = \sqrt{x}$. $\displaystyle\int_0^{a^2} \sqrt{1 + \frac{1}{4x}}\, dx = \int_0^a \sqrt{1 + 4y^2}\, dy$, for any positive number a.

3. $\displaystyle\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x = e^2$.

4. $\displaystyle\frac{d}{dx}\left(8^x\right) = x8^{x-1}$.

5. $\cos\left(\tan^{-1} x\right) = \dfrac{x}{\sqrt{1+x^2}}$.

6. $\displaystyle\int_0^6 \frac{x}{x^2-4}\, dx = \ln\left(2\sqrt{2}\right)$.

7. $\dfrac{x^2+4}{x(x^2-4)}$ can be put in the form $\dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{x-2}$.

8. $\ln(\ln x) = o(\ln x)$.

9. A function has an inverse if and only if it is increasing or decreasing.

10. $\sin^{-1}\left(\sin\dfrac{3\pi}{4}\right) = \dfrac{3\pi}{4.}$

(下頁還有試題)

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. If $f(x) = \begin{cases} \dfrac{1-\cos x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then find $f'(0)$.

Answer：_____.

2. Let $y = \sqrt[3]{\dfrac{x(x+2)}{x^2+1}}$. Use logarithmic differentiation to find $\dfrac{dy}{dx}\Big|_{x=1}$.

Answer：_____.

3. Let $y = \displaystyle\int_{e^{\sqrt{x}}}^{e^x} \ln t \, dt$, $x > 0$. Find $\dfrac{dy}{dx}$.

Answer：_____.

4. Order(排序) the following functions from slowest growing to fastest growing as $x \to \infty$. **a.** $\sqrt{x}$ **b.** $\ln x^3$ **c.** x^x **d.** e^x

Answer：_____.

5. Let $f(x) = x^3 + 3\sin x + 2\cos x$. Find the value of $\dfrac{df^{-1}}{dx}$ at the point $x = 2 = f(0)$.

Answer：_____.

6. Evaluate $\displaystyle\int_0^1 x\sqrt{1-x}\, dx$.

Answer：_____.

7. Evaluate $\displaystyle\int_{\pi/3}^{\pi/2} \frac{\sin^2 x}{\sqrt{1-\cos x}}\, dx$.

Answer：_____.

8. Evaluate $\displaystyle\int_{-\pi}^{\pi} \sin 3x \sin 3x \, dx$.

Answer：_____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 3$, about the x-axis.

2. (*10* points) Evaluate the integrals.

a. $\int \sec^3\theta \, d\theta$

b. $\int \sqrt{x^2 - 9} \, dx$

3. (*10* points) Evaluate the integrals.

a. $\int \dfrac{x^3 e^{x^2}}{(x^2+1)^2} \, dx$

b. $\int_1^e x^3 \ln x \, dx$

4. (*10* points) Find the limits.

a. $\lim\limits_{x\to\infty} x^{1/\ln x}$

b. $\lim\limits_{x\to 0} \dfrac{\tan^{-1} x^2}{x \sin^{-1} x}$

5. (*10* points) Evaluate the integral.

$$\int \frac{x^2 - x + 2}{x^3 - 1} \, dx$$

6. (*10* points) Investigate the convergence:

a. $\int_0^\infty \dfrac{dx}{\sqrt{x^6+1}}$

b. $\int_1^\infty \dfrac{1 - e^{-x}}{x} \, dx$

(試題結束)