

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. There is a function f for which f is not continuous at 5 but f' exists at 5.
2. If f is continuous on $[a, b]$ and $f'(c) = 0$ for some $c \in [a, b]$, then f has an absolute extremum at c .
3. The accumulated amount corresponding to invest \$180,000 at 2%/year compounded monthly is larger than that corresponding to compounded quarterly.
4. Suppose f is nonpositive (i.e., $f(x) \leq 0$) and continuous on $[a, b]$, then the area of the region below the x -axis and above the graph of f on $[a, b]$ is given by
$$\int_a^b f(x) dx.$$
5.
$$\int_0^3 x\sqrt{9-x^2} dx = \frac{1}{2} \int_0^9 \sqrt{x} dx.$$
6. The producers' surplus is given by $PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x) dx$, where $S(x)$ is the supply function, \bar{p} is the unit market price, and \bar{x} is the quantity supplied.
7. If $f(x) = \ln a^x$ then $f'(x) = \ln a$, where $a > 0$ is a constant.
8. Let f be a continuous function defined on $[a, b]$, then
$$\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$$
 for $a < x < b$.

(下頁還有試題)

9. $\int_a^b f(x)g(x)dx = \left[\int_a^b f(x)dx \right] \left[\int_a^b g(x)dx \right]$ if f and g are integrable on $[a, b]$.

10. $\int_1^2 (1+x^3)dx = \int_1^3 (1+x^3)dx + \int_3^2 (1+x^3)dx$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Suppose that $\lim_{x \rightarrow 3} f(x) = 2$, $\lim_{x \rightarrow 2} g(x) = -2$, $\lim_{x \rightarrow 3} h(x) = 2$ and $g'(3) = -1$, find

$$\lim_{x \rightarrow 3} \frac{g(f(x)) + x^2}{h^2(x)}.$$

Answer: _____.

2. Find the interest rate needed to for an investment of \$5000 to double in 8 years if interest is compounded continuously.

Answer: _____.

3. Find the indefinite integral $\int \left(\frac{\ln \sqrt{x}}{x} + \frac{e^x}{e^x + 3} \right) dx$.

Answer: _____.

4. The quantity demanded x (in units of a hundred) of the Mikako miniature cameras per week is related to the unit price p (in dollars) by

$$p = D(x) = -0.2x^2 + 80.$$

Suppose that the market price is \$60 per unit, find the consumers' surplus.

Answer: _____.

5. Find an equation of the tangent line to the graph of $f(x) = xe^{-x/3}$ at the point where f attains its relative maximum.

Answer: _____.

6. Find the derivative of the function $f(x) = (\ln x)^x$.

Answer: _____.

(下頁還有試題)

7. In calm water, the oil spilling from the ruptured hull of a grounded tanker forms an oil slick that is circular in shape. If the radius r of the circle is increasing at the rate of $r'(t) = \frac{30}{\sqrt{2t+4}}$ feet/minute t min after the rupture occurs, find an expression for the radius at any time t . (Hint: $r(0) = 0$.)

Answer: _____.

8. Let $f(x) = \sqrt{x}$. Find x so that $f(x)$ is equal to the average value of f over the interval $[0, 4]$.

Answer: _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) A car leaves an intersection traveling west. Its position 4 sec later is 20 ft from the intersection. At the same time, another car leaves the same intersection heading north so that its position t sec later is $t^2 + 2t$ ft from the intersection. If the speed of the first car 4 sec after leaving the intersection is 9 ft/sec, find the rate at which the distance between the two cars is changing at that instant of time.
2. (10 points) A certain city's rate of electricity consumption is expected to grow exponentially with a growth constant of $k = 0.05$. If the present rate of consumption is 30 million kilowatt-hours (kWh) per year, what should be the total production of electricity over the next 2 years in order to meet the projected demand?
3. (10 points) Camille purchased a 15-year franchise for a computer outlet store that is expected to generate income at the rate of $R(t) = 400,000$ dollars/year. If the prevailing interest rate is 8%/year compounded continuously, find the present value of the franchise.

(下頁還有試題)

4. (10 points) Garland Mills purchased a certain piece of machinery 3 years ago for \$500,000. Its present sale value is \$400,000. Assuming the machine's resale value decreases exponentially, what will it be 6 years from now?
5. (10 points) Evaluate the definite integrals.
- a. $\int_1^e \frac{1}{x\sqrt{1+\ln x}} dx$
- b. $\int_1^2 \left(2e^{-4x} + \frac{1}{x^2} \right) dx$
6. (10 points) Sketch the graph, and find the area of the region completely enclosed by the graphs of the functions $f(x) = x^3 - 6x^2 + 9x$ and $g(x) = x^2 - 3x$.

(試題結束)