

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $\sum_{n=1}^{\infty} a_n x^n$ is divergent at $x = -1$, then it is divergent at $x = 2$.
2. The series $\sum_{n=0}^{\infty} \cos n\pi$ is convergent.
3. The series $\sum_{n=1}^{\infty} (\sqrt[n]{3} - 1)^n$ is convergent.
4. If $\sum_{n=1}^{\infty} a_n$ is convergent, and if $a_n > 0$ and $a_n \neq 1$ for all n , then $\sum_{n=1}^{\infty} \frac{a_n}{1 - a_n}$ is also convergent.
5. If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then $\sum_{n=1}^{\infty} a_n^2$ is convergent.
6. $\lim_{n \rightarrow \infty} \left(\frac{1}{n}\right)^{\frac{1}{\ln(n)}}$ converges to zero.
7. If the series $\sum_{n=0}^{\infty} a_n (x - 3)^n$ converges for $x = 2$, then the series $\sum_{n=0}^{\infty} a_n$ converges.
8. If $a_n > 0$ and $\frac{a_{n+1}}{a_n} < 1$ for all $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ is convergent.
9. The graph of $r^2 = 4r \sin \theta$ is a circle with radius 2 and Cartesian coordinates center $(0, 2)$.
10. If $a_n > 0$ for all $n \in \mathbf{N}$ and $\lim_{n \rightarrow \infty} a_n = \frac{1}{2}$, then the series $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges.

(下頁還有試題)

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the Taylor series generated by $f(x) = \frac{1}{x^2}$ at $x = 3$.

Answer : _____.

2. For what values of p is the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}$ converges.

Answer : _____.

3. For what values of p is the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

convergent? Answer : _____.

4. You drop a ball from 5 meters above a flat surface. Each time the ball hits the surface after a distance h , it rebounded a distance $\frac{h}{3}$. Find the total distance the ball travels up and down. Answer : _____.

5. Find the area under one arch of the cycloid $x = t - \sin(t)$ and $y = 1 - \cos(t)$.

Answer : _____.

6. Find the first four nonzero terms of the binomial series for the function

$$\frac{1}{(1-x^2)^{1/3}}.$$

Answer : _____.

7. Find the polar equation for the circle $x^2 + (y-3)^2 = 9$.

Answer : _____.

8. Find the interval of convergence for

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n-1}}{2n-1}.$$

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Determine convergence or divergence:

a. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n \ln n}}$

b. $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$

2. (10 points)

$$\sum_{n=2}^{\infty} \frac{(2x+3)^{2n+1}}{\ln n}$$

a. Find the series' radius of convergence and the interval of convergence.

b. For what values of x does the series converge absolutely?

c. For what values of x does the series converge conditionally?

3. (10 points) Find the surface area when the curve

$$x = \ln(\sec t + \tan t) - \sin t$$

$$y = \cos t, \quad 0 \leq t \leq \frac{\pi}{3}$$

is revolved about the x -axis.

4. (10 points) A curve C is defined by the parametric equations

$$x = \cos^3 t, \quad y = \sin^3 t, \quad 0 \leq t \leq 2\pi.$$

a. Find the equation of the tangent line at $t = \frac{\pi}{4}$.

b. Find $\frac{d^2 y}{dx^2}$ if $\frac{dx}{dt} \neq 0$.

5. (10 points) Evaluate

a. $\sum_{n=1}^{\infty} \frac{1}{1+2+3+\dots+n}$

b. $3 + \ln 3 + \frac{(\ln 3)^2}{2!} + \dots + \frac{(\ln 3)^n}{n!} + \dots$

6. (10 points) Suppose that $\sum_{n=1}^{\infty} 2^n a_n$ converges. Prove and disprove that $\sum_{n=1}^{\infty} |a_n|$ converges.

(試題結束)