管院微積分聯合教學　　會考四　　**Mar 22, 2016**

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。 (不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If the data consist of two distinct points, then the least square line is just the line that passes through the two points.

2. If f_{xx} and f_{yy} have the same signs at a critical point (a, b) of f, then f has a saddle point at (a, b).

3. The domain of $f(x, y) = \dfrac{1}{x^2 + y}$ is the whole real line.

4. The formula for the volume of a right circular cone of radius r and height h can be derived by revolving a triangle either about the x-axis or y-axis.

5. $\displaystyle\int \frac{1}{x \ln(x)}\, dx = \ln|\ln(x)| + constant.$

6. Integration by parts comes from the product rule of differentiation.

7. An approximation of $\displaystyle\int_3^5 x^2 + 3x + 1 dx$ by using Simpson's rule with $n = 10$ is equal to the value of $\displaystyle\int_3^5 x^2 + 3x + 1 dx$.

8. $\displaystyle\int_{-\infty}^{\infty} (1 + |x|)^{-1} dx$ converges.

9. Suppose that $f(x, y) = x^2 y + e^{xy}$, then two level curves $f(x, y) = 2$ and $f(x, y) = 3$ don't intersect.

10. If (a, b) is a critical point of f, then f must have a relative maximum or minimum at (a, b).

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Suppose the definite integral $\int_a^b f(x)\,dx$ is approximated with n subintervals, tha maximum error incurred in using the Trapezoidal Rule is $\frac{M(b-a)^3}{12n^2}$, where M is a number such that _____ for all x in $[a, b]$.

Answer:_____.

2. The domain of the function $f(x, y) = \frac{1}{x} + \frac{1}{x-y} - e^{x+y}$ is _____.

Answer:_____.

3. The second partial derivatives f_{xy} and f_{yx} are equal if _____.

Answer:_____.

4. Find an equation of the least-squares line for the data:

$$(1, 1),\ (2, 1),\ (3, 2),\ (4, 4) \text{ and } (5, 4).$$

Answer:_____.

5. Evaluate the integral

$$\int_1^2 x \ln x dx.$$

Answer:_____.

6. Suppose that the demand for commodity A is given by

$$x = f(p, q) = (p + q)^{-2},\ p > 0,\ q > 0,$$

and the demand for commodity B is given by

$$y = g(p, q) = (p + q)^{-3},\ p > 0,\ q > 0,$$

where p and q denote the prices of commodity A and B respectively. Determine whether these two commodities are substitute, complementary, or neither.

Answer:_____.

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7. Mark invests 500 dollars per month in perpetuity and his investment earns interest at the rate of 5% per year compounded continuously. What is the present value of his investment.

Answer:_____.

8. The integral

$$\int_1^\infty \frac{1}{x^p} dx$$

is convergent for which p.

Answer:_____.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Ross-Simons Company has a monthly advertising budget of $\$60,000$. Their marketing department estimates that if they spend x dollars on newpaper advertising and y dollars on television advertising, then the monthly sales will be given by

$$f(x, y) = 90x^{1/4}y^{3/4}$$

dollars. Determine how much money Ross-Simons should spend on newspaper ads and on television ads each month to maximize its monthly sales. (In order to get credits, you must apply the method of Lagrange multipliers.)

2. (*10* points) The rate of reaction to a drug is given by $r'(x) = 2x^2e^{-x}$, where x is the number of hours since the drug was administered. Find the total reaction to the drug from $x = 1$ to $x = 6$.

3. (*10* points) Approximate the value of

$$\int_0^1 x^3 \, dx$$

using the Simpson's rule with $n = 6$ and find an error.

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4. (*10* points) Evaluate

$$\int_0^\infty x^3 e^{-x^2}\, dx.$$

Show **ALL** your work or lose points.

5. (*10* points) Find the volume of the solid obtained by revolving the region bounded by the curves $y = e^{x^2}$ and $y = e^{2-x}$ about the x-axis. Write the formula **ONLY. DO NOT** calculate it.

6. (*10* points) Find the points that the function

$$f(x, y) = x^2 + 3y^2 + 2xy - 2x + 3y + 5$$

has relative extrema at? **Ill-explanation causes 0 point**.

(試題結束)