微積分聯合教學　會考五　**May 3, 2016**

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Given nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, we have $(\mathbf{u}\times\mathbf{v})\cdot\mathbf{u} = (\mathbf{u}\times\mathbf{v})\cdot\mathbf{v}$.

2. The curvature is constant for straight lines and circles.

3. There exists a function f with continuous second-order partial derivatives such that $f_x(x,y) = xy$ and $f_y(x,y) = xy$.

4. There is a direction $\mathbf{u}$ such that the directional derivative of $f(x,y) = x^2 - 3xy + 4y^2$ at the point $(1,2)$ equals 14.

5. Let $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$ be a smooth vector function. If $|\mathbf{r}(t)| = 2$ for all t, then $|\mathbf{r}(t)\times\mathbf{r}'(t)| = |\mathbf{r}'(t)|$.

6. If $|\mathbf{r}(t)| = 1$ for all t, then $|\mathbf{r}'(t)|$ is a constant.

7. Let $(D_{\mathbf{u}}f)_{P_0}$ denote the derivative of the function f at the point P_0 in the direction $\mathbf{u}$. Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be any unit vector such that $\mathbf{u}\neq-\mathbf{v}$ and $\mathbf{w} = \dfrac{\mathbf{u}+\mathbf{v}}{|\mathbf{u}+\mathbf{v}|}$. If $f(x,y)$ is differentiable in an open region containing P_0, then $|\mathbf{u}+\mathbf{v}|(D_{\mathbf{w}}f)_{P_0} = (D_{\mathbf{u}}f)_{P_0} + (D_{\mathbf{v}}f)_{P_0}$.

8. Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be any vector in $\mathbf{R}^3$. If $\mathbf{w}\times\mathbf{u} = \mathbf{w}\times\mathbf{v}$ and $\mathbf{w}\neq\mathbf{0}$, then $\mathbf{u} = \mathbf{v}$.

9. The curve $\mathbf{r}(t) = t^3\mathbf{i} + 2t^3\mathbf{j} + 5t^3\mathbf{k}$ is a line in $\mathbf{R}^3$.

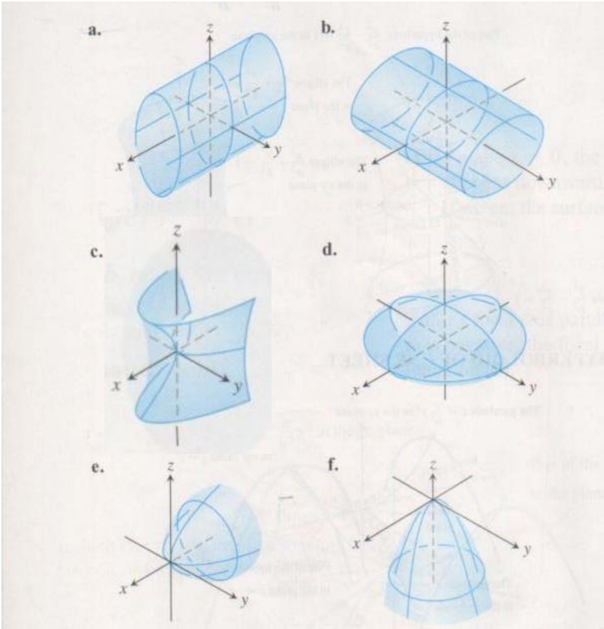
(下頁還有試題)

10. The function $f(x, y) = 4 - (x-1)^2 - (y-1)^2$ reaches its *absolute maximum* inside the triangular region with vertices $(0, 0)$, $(5, 0)$, and $(0, 4)$.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let G_1, G_2, G_3, G_4 be the graphs corresponding to the following equations (E_1) $x^2 + 2z^2 = 8$ (E_2) $x^2 + y^2 + 4z^2 = 10$ (E_3) $x = z^2 - y^2$ (E_4) $x = -y^2 - z^2$, respectively. Then find (G_1, G_2, G_3, G_4).



Answer：____________.

2. Let $z^3 - xy + yz + y^3 - 2 = 0$. Find the value of $\dfrac{\partial z}{\partial x}$ at the point $(1, 1, 1)$.

Answer：____________.

3. Find the saddle point of the function $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

Answer：____________.

4. Find the length of the indicated portion of the curve

$$\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j} + \sqrt{5}t\mathbf{k}, \quad 0 \le t \le 2\pi.$$

Answer：____________.

5. Find the direction in which $f(x, y, z) = (x/y) - yz$ decreases most rapidly at $P_0(4, 1, 1)$. Answer：____________.

(下頁還有試題)

6. Find the volume of the box (parallelepiped) determined by the vectors $\overrightarrow{OP}$, $\overrightarrow{OQ}$, and $\overrightarrow{OR}$, where $P = (2, -2, 1)$, $Q = (3, -1, 2)$, $R = (3, -1, 1)$ and O is the origin $(0, 0, 0)$ in the space $\mathbf{R}^3$. Answer：___________.

7. Find the maximum value of the function $f(x, y) = 5x + 12y$ on the circle $x^2 + y^2 = 1$. Answer：___________.

8. Let D be the domain of the two-variable function $f(x, y) = \dfrac{1}{xy}$. Which of the following statements are true?

(A) The point $(0, 1) \in D$.

(B) The point $(0, 1)$ is an interior point of D .

(C) The point $(0, 1)$ is a boundary point of D .

(D) The region D is open.

(E) The region D is closed.

(F) The region D is neither open nor closed.

Answer：___________.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) Find $\mathbf{T}$, $\mathbf{N}$, and κ for the space curve

$$\mathbf{r}(t) = (3\sin t)\mathbf{i} + (3\cos t)\mathbf{j} + 4t\mathbf{k}.$$

2. (*10* points) Let

$$f(x, y) = \begin{cases} 0 & , xy \neq 0 \\ 1 & , xy = 0 \end{cases} .$$

a. Find each of the partial derivatives $\dfrac{\partial f}{\partial x}(0, 0)$, $\dfrac{\partial f}{\partial x}(0, 1)$, $\dfrac{\partial f}{\partial y}(0, 0)$ and $\dfrac{\partial f}{\partial y}(0, 1)$ if it exists, or show that it does not exist by definition.

b. Prove that f is not differentiable at the origin.

(下頁還有試題)

3. (*10* points) Find the point of intersection of the lines $x = 2t + 1$, $y = 3t + 2$, $z = 4t + 3$, and $x = s + 2$, $y = 2s + 4$, $z = -4s - 1$, and then find the plane determined by these lines.

4. (*10* points) Show that the function

$$f(x, y) = \frac{x^4 - y^2}{x^4 + y^2}$$

has no limit as $(x, y) \to (0, 0)$.

5. (*10* points) The cylinder $x^2 + y^2 - 1 = 0$ and the plane $x + y + z - 1 = 0$ meet in an ellipse E. Find parametric equations for the line tangent to E at the point $P_0\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1 - \sqrt{2}\right)$.

6. (*10* points) Find the vector function $\mathbf{r}(t)$ such that

$$\frac{d\mathbf{r}}{dt} = \frac{3}{2}\sqrt{t+1}\mathbf{i} + e^{-t}\mathbf{j} + \frac{1}{t+1}\mathbf{k} \text{ for } t > 0$$

and $\mathbf{r}(0) = \mathbf{k}$.

(試題結束)