

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機及任何通訊器材，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. The area of a region R in the xy -plane is the double integral of the constant function $f(x, y) = 1$ over R .
2. The functions $y = 0$ and $y = C$ are solutions to the differential equation $y' = y(C - y)f(x)$ for any function f of x .
3. Suppose that y is a solution to the differential equation $y' = 3(100 - 2y)$, then $\lim_{x \rightarrow \infty} y(x) = 50$.
4. The integral of $f(x, y)$ over the region $\mathbf{R} = \{(x, y) | x^2 \leq y \leq x; 0 \leq x \leq 1\}$ is $\int_0^1 \int_y^{\sqrt{y}} f(x, y) dx dy$.
5. If f is a probability density function on $[a, b]$, then $0 \leq f \leq 1$ on $[a, b]$.
6. Let X be the random variable with the normal probability density function $e^{-\frac{1}{2}(x+3)^2} / \sqrt{2\pi}$, then $P(X > 1) > 1/2$.
7. The population density (number of people per square mile) of a city is described by the function

$$f(x, y) = \frac{10000e^y}{1 + 0.5|x|} \quad (-10 \leq x \leq 10; -30 \leq y \leq 0)$$

where x and y are measured in miles. Then the population inside the region $R = \{(x, y) | x^2 - 25 \leq y \leq 0; -5 \leq x \leq 5\}$ is given by $2 \int_{-5}^0 \int_{x^2-25}^0 \frac{10000e^y}{1 - 0.5x} dy dx$.

8. Suppose that money deposited into a bank grows at a rate that is proportional to the amount accumulated. If the amount on deposit initially is P , then the accumulated amount A after t years is governed by the differential equation

$$\frac{dA}{dt} = kA \quad A(0) = P$$

where k is a constant.

9. Use Euler's method with $n = 6$ to obtain an approximation of the solution of the initial-value problem $y' = x - 2y$ $y(0) = 1$ when $x = 2$. It is known that $y(\frac{5}{3}) \approx \frac{143}{243}$. Then

$$y(2) \approx \frac{143}{243} + \frac{1}{3} \left(\frac{5}{3} - \frac{286}{243} \right).$$

10. Let $f(x, y) = xy$ be the joint probability density function for the random variables X and Y on $D = \{(x, y) | 0 \leq x \leq 2; 0 \leq y \leq 1\}$. Then $P(X + Y \leq 2) = \int_0^2 \int_0^{2-x} xy \, dy dx$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the general solution to the differential equation $\frac{dy}{dx} = \frac{y}{x}$.

Answer: _____.

2. Let f be the uniform density function on the interval $[0, a]$. Find the standard deviation of the associated random variable.

Answer: _____.

3. The median of a random variable X is defined to be the number m such that $P(X < m) = \frac{1}{2}$. Find the median of the random variable X with the probability density function.

$$f(x) = \frac{1}{2}e^{-x/2}, 0 \leq x < \infty.$$

Answer: _____.

4. The production of a South American country is given by the function

$$f(x, y) = 20x^{\frac{3}{4}}y^{\frac{1}{4}}$$

when x units of labor and y units of capital are utilized. Find the approximate change in output if the amount expended on labor is increased from 256 to 258 units and the amount expended on capital is decreased from 16 to 15 units.

Answer: _____.

5. Evaluate the double integral $\int \int_R \frac{x}{y^2} e^{\frac{x}{y}} dA$, where R is the rectangle defined by $0 \leq x \leq 1$ and $\frac{1}{2} \leq y \leq 1$.

Answer: _____.

6. Suppose glucose is infused into the bloodstream at a constant rate of C g/min and at the same time the glucose is converted and removed from the bloodstream at a rate proportional to the amount of glucose present with proportionality k . Find a differential equation, but do not solve, to describe the amount of glucose $A(t)$ present in the bloodstream at any time t .

Answer: _____.

7. The life span of a certain light bulb (in hours) is an exponentially distributed random variable X with expected value of 1000 (hours). What is the probability of a light bulb selected at random that will survive at least 700 hours?

Answer: _____.

8. The grade point average (GPA) of the senior class of a certain high school is normally distributed with a mean of 2.8 and a standard deviation of 0.3. If a senior in the top 12% of his or her class is eligible for the admission to any of the nine campuses of the state university system, what is the minimum GPA that a senior should have to ensure eligible for university admission? Round the answer to two decimal places.

(Hint: $P(Z < -1.17) \approx 0.12$, where Z is the standard normal random variable.)

Answer: _____.

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Use a double integral to find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y = 1$ and the surface $z = 4 - x^2 - y^2$.

2. (10 points) Consider the iterated integral

$$\int_0^3 \left[\int_{x^2}^9 x e^{y^2} dy \right] dx.$$

Sketch the region R over which the function $x e^{y^2}$ is integrated, then evaluate the integral. (Hint: change the order of integration.)

3. (10 points) In a certain chemical reaction, a substance is converted into another substance at a rate proportional to the square of the amount of the first substance at any time t . Initially ($t = 0$), 50 g of the first substance was present; 1 hr later, only 10 g of it remained. Find an expression that gives the amount of the first substance at any time t . What is the amount present after 2hr?

4. (10 points) Find the average value of the function $f(x, y) = \ln x$ over the plane region R bounded by the graphs of $y = 3x$ and $y = 0$ from $x = 1$ to $x = 2$.

5. (10 points) A tank initially contains 60 gal of brine, in which 20 lb of salt is dissolved. Brine containing 3 lb of dissolved salt per gallon flows into the tank at the rate of 3 gal/min, and the well-stirred mixture flows out of the tank at the same rate. How much salt is present in the tank at the end of 20 min?

6. (10 points) The life expectancy (in years) of a certain brand of plasma TV is a continuous random variable with probability density function

$$f(t) = 7(7 + t^2)^{-3/2} \quad (0 \leq t < \infty)$$

How long is one of these plasma TVs expected to last?

(試題結束)