微積分聯合教學　會考六　**Jun 14, 2016**

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

**1.** If $a_n > 0$ and $\dfrac{a_{n+1}}{a_n} < 1$ for all $n \geq 1$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

**2.** There is a direction $\mathbf{u}$ such that the directional derivative of $f(x, y) = x^2 - 3xy + 4y^2$ at the point $(1, 2)$ equals 14.

**3.** Let $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$ be a smooth vector function. If $|\mathbf{r}(t)| = 2$ for all $t$, then $|\mathbf{r}(t) \times \mathbf{r}'(t)| = |\mathbf{r}'(t)|$.

**4.** The line integral of the constant function $f(x, y, z) = -5$ over a smooth curve $C$ of length 6 is $-30$.

**5.** If a simple closed curve $C$ in the plane and the region $R$ it encloses satisfy the hypotheses of Green's Theorem, then $\oint_C \sin^2 x\, dx + (x + e^{y^2})\, dy =$Area of $R$.

**6.** $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 + y^2\; dydx = \frac{\pi}{6}$.

**7.** If $f(x, y, z)$ has continuous first order partial derivatives on $R^3$ and $C$ is a smooth path from the point $(a_1, a_2, a_3)$ to $(b_1, b_2, b_3)$ then $\displaystyle\int_C \nabla f \cdot d\mathbf{r} = f(b_1, b_2, b_3) - f(a_1, a_2, a_3)$.

**8.** $\displaystyle\int_0^1 \int_0^x (3 - x - y)\; dydx = \int_0^x \int_0^1 (3 - x - y)\; dxdy$.

(下頁還有試題)

**9.** $\displaystyle\int_{-1}^{1}\int_{0}^{1} ye^{x^2+y^2}\,dxdy = 0.$

**10.** If $\displaystyle\int_C \mathbf{F}\cdot\mathbf{T}\,ds = 0$, then $\mathbf{F}$ and $\mathbf{T}$ are orthogonal on each point along the path $C$.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

**1.** Let $D$ be the domain of the two-variable function $f(x,y) = \dfrac{1}{xy}$. Which of the following statements are true?

(A) The point $(0,1) \in D$ .

(B) The point $(0,1)$ is a boundary point of $D$.

(C) The point $(0,1)$ is an interior point of $D$.

(D) The region $D$ is closed.

(E) The region $D$ is open.

(F) The region $D$ is neither open nor closed.

Answer：___________.

**2.** Convert the integral

$$\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} x\,dzdydx$$

to an equivalent integral in spherical coordinates. (**Do not evaluate the integral.**)

Answer：___________.

**3.** Solve the system $u = 3x + 2y$, $v = x + 4y$ for $x$ and $y$ in terms of $u$ and $v$, Then find the value of the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$.

Answer：___________.

**4.** Evaluate $\displaystyle\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz.$

Answer：___________.

**5.** Find the work done by the force field $\mathbf{F} = (x - y^2)\mathbf{i} + (y - z^2)\mathbf{j} + (z - x^2)\mathbf{k}$ that moves an object along the line segment from $(0, 0, 1)$ to $(1, 1, 0)$.

Answer：___________.

**6.** Find the flux of $\mathbf{F} = (x - y)\mathbf{i} + (x)\mathbf{j}$ across the circle $x^2 + y^2 = 1$ in the $xy$-plane.

Answer：___________.

**7.** Convert $\int_0^{\pi/4} \int_0^{2\sec\theta} r^5 \sin^2\theta \; dr d\theta$ to an equivalent Cartesian integral. (**Do not evaluate the integral.**)

Answer：___________.

**8.** Let $C$ be the square cut from the first quadrant by the lines $x = 1$ and $y = 1$. Evaluate $\oint_C xydy - y^2dx$.

Answer：___________.

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

**1.** (*10* points) Find the area under one arch of the cycloid $x = t - \sin(t)$ and $y = 1 - \cos(t)$.

**2.** (*10* points) Sketch the region of integration, reverse the order of integration, and evaluate the integral.

$$\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2} \, dxdy.$$

**3.** (*10* points) Using polar integration to evaluate

$$\int\int_R e^{x^2+y^2} \; dydx,$$

where $R$ is the semicircular region bounded by the $x$-axis and the curve $y = \sqrt{1 - x^2}$.

(下頁還有試題)

**4.** (*10* points)

**a.** Show that $\mathbf{F}(x, y, z) = (y \sin z)\mathbf{i} + (x \sin z)\mathbf{j} + (xy \cos z)\mathbf{k}$ is a conservative vector field.

**b.** Find a function $f$ such that $\mathbf{F} = \nabla f$.

**5.** (*10* points) Evaluate the integral $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^{3} (x^2 + y^2)\, dzdydx$ by converting it into an integral in cylindrical coordinates.

**6.** (*10* points) Evaluate the integral

$$\int\int_R (2x^2 - xy - y^2)\, dxdy$$

for the region $R$ in the first quadrant bounded by the lines $y = -2x+4$, $y = -2x+7$, $y = x - 2$, and $y = x + 1$. (Hint: Let $u = x - y$ and $v = 2x + y$.)

(試題結束)