管院微積分聯合教學　　會考六　　　　**Jun 14, 2016**

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。 (不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f_{xx} and f_{yy} have the same signs at a critical point (a, b) of f, then f has a saddle point at (a, b).

2. $\displaystyle\int_{-\infty}^{\infty} (1 + |x|)^{-1} dx$ converges.

3. Suppose that $f(x, y) = x^2 y + e^{xy}$, then two level curves $f(x, y) = 2$ and $f(x, y) = 3$ don't intersect.

4. The functions $y = 0$ and $y = C$ are solutions to the differential equation $y' = y (C - y) f(x)$ for any function f of x.

5. If f is a probability density finction on $[a, b]$, then $0 \le f \le 1$ on $[a, b]$.

6. If $\{a_n\}$ and $\{b_n\}$ are sequences such that $\displaystyle\lim_{n \to \infty} (a_n + b_n)$ exists, then both $\displaystyle\lim_{n \to \infty} a_n$ and $\displaystyle\lim_{n \to \infty} b_n$ must exist.

7. If $\displaystyle\lim_{n \to \infty} a_n = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.

8. If $\displaystyle\sum_{n=0}^{\infty} a_n(x - a)^n$ has radius of covergence R, then $\displaystyle\sum_{n=0}^{\infty} na_n(x - a)^n$ has radius of covergence R.

9. If $|r| > 1$, then $\displaystyle\sum_{n=1}^{\infty} \frac{1}{r^n} = \frac{1}{r - 1}$.

10. The graph of $f(x) = \sin x + \cos x$ is concave upward on the interval $\left(0, \frac{\pi}{2}\right)$.

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填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. The integral
$$\int_1^\infty \frac{1}{x^p} dx$$
   is convergent for which p.

   Answer:_____.

2. Suppose glucose is infused into the bloodstream at a constant rate of C g/min and at the same time the glucose is converted and removed from the bloodstream at a rate proportional to the amount of glucose present with proportionality k. Find a differential equation, but do not solve, to describe the amount of glucose $A(t)$ present in the bloodstream at any time t.

   Answer:_____.

3. Find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{n^2 + 3n + 2}$.

   Answer:_____.

4. Determine the sixth Taylor polynomial of $\cos x$ at $x = 0$.

   Answer:_____.

5. It can be shown that the function $f(x) = x^3 - x - 1$ has a zero between $x = 1$ and $x = 2$. One can use Newton's method to get a sequence $x_0, x_1, x_2, ...$ to approximate the indicated zero of f. Suppose $x_0 = 1$, find x_2.

   Answer:_____.

6. Evaluate $\lim_{x \to 0} \frac{x^3}{x - \tan x}$. Answer:_____.

7. Find the area of the region under the graph of $f(x) = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$. Answer:_____.

8. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(-3x)^{2n}}{9^n(n+1)}$.

   Answer:_____.

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計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) The Robinson family wishes to establish a scholarship fund at a collage. If a scholarship in the amount of \$10000 is to be awarded on an annual basis beginning next year, find the amount of the endowment they are required to make now. Assume that this fund will earn interest at a rate of 5% per year compounded continuously.

2. (*10* points)

   a. Use the integral test to determine whether the series $\sum_{n=2}^{\infty} \dfrac{1}{n(\ln n)^3}$ is convergent or divergent.

   b. Use the comparison test to determine whether the series $\sum_{n=2}^{\infty} \dfrac{1}{n^{2/3} - 1}$ is convergent or divergent.

3. (*10* points) It is known that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + ... + x^n + ... \quad (-1 < x < 1).$$

   Let $f(x) = \dfrac{1}{1 + 2x}$.

   a. Find the power series representation of $f(x)$ at $x = 2$.

   b. Find the power series representation of $f'(x)$ at $x = 2$.

4. (*10* points) The cross section of a drain is a trapezoid, as shown in the figure. The sides and the bottom of the trapezoid are each 10 ft long. Determine the angle t such that the drain will have a maximal cross-sectional area.
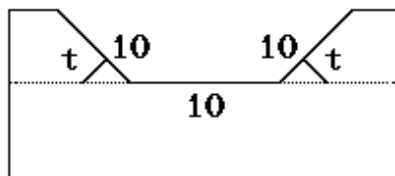
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5. (*10* points) The revenue of McMenamy's Fish Shanty, located at a popular summer resort, is approximately

$$R(t) = 2\left(5 - 4\cos\frac{\pi t}{6}\right) \qquad (0 \le t \le 12)$$

during the tth week ($t = 0$ corresponds to the first day of June), where R is measured in thousands of dollars. What is the total revenue realized by the restaurant over the 12-week period starting June 1?

6. (*10* points) It has been estimated that service industries, which currently make up 30% of the nonfarm workforce in a certain country, will continue to grow at the rate of

$$R(t) = 6e^{1/(2t+1)}$$

percent per decade t decades from now. Estimate the percentage of the nonfarm workforce in service industries two decades from now.

(Hint: Apply the second Taylor polynomial.)

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