

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Let f and g be two functions, then $\frac{d}{dx}f(g(x)) = \frac{d}{dx}g(f(x))$.
2. In the xy -plane, every linear equation represents a straight line.
3. If f is a continuous function on an open interval (a, b) and if $f(a)$ and $f(b)$ have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b) .
4. If the demand is elastic at p , then an increase in the unit price will cause the revenue to increase.
5. We say that f is continuous on the interval (a, b) if f is continuous at every point in (a, b) .
6. Consider the equation: $x^2 + y^2 = 1$. Then y is a function of x .
7. Suppose that L_1 and L_2 are two lines in the plane with slope $m_1 \neq 0$ and $m_2 \neq 0$ respectively. If $m_1 \neq m_2$, then L_1 and L_2 intersect exactly one point.
8. If f is differentiable, then $\frac{d}{dx} \left[\frac{f(x)}{x^2 + 1} \right] = \frac{f'(x)}{2x}$.
9. $\lim_{x \rightarrow \infty} \frac{(x^{50} + 1)^2}{x^{100} + 2} = 1$.

(下頁還有試題)

10. Let $f(x) = \begin{cases} \frac{|x|}{x} & , x \neq 0 \\ k & , x = 0 \end{cases}$. We can choose some value k such that the function f is continuous at $x = 0$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the domain of

$$f(x) = \frac{\sqrt{x-3}}{x^2-4}.$$

Answer: _____.

2. We define $f(x) = \begin{cases} x-2 & , x < 1; \\ x^2-2x & , 1 \leq x \leq 3; \\ 2 & , x > 3. \end{cases}$

Find all points that $f(x)$ is not continuous at. If there is no such point, please put NONE in your answer sheet. Answer: _____.

3. Consider the demand equation $f(p) = (1+p)^{-1}$, $p \geq 0$. Find the elasticity of demand at price $p = 20$. Answer: _____.

4. Find the value of

$$\lim_{x \rightarrow \infty} \frac{3x^{9/2} + 1000x^4}{x^5 + x^4 + 10000}.$$

Answer: _____.

5. David wanna paint a right circular cylinder with radius r meters and height $3r$ meters. It costs 10 dollars to paint 1 square meter of an area. How much money does it cost to paint all the surface of the right circular cylinder?

Answer: _____.

6. Let $f(x) = \begin{cases} k(x+1)^2 & , x \leq 0 \\ x+2 & , x > 0 \end{cases}$. Find the value of k that will make f continuous on $(-\infty, \infty)$. Answer: _____.

7. Let $F(x) = x^4 - 32x$. Find the point on the graph of F where the tangent line is horizontal. Answer: _____.

8. Let $h(x) = \frac{f(x)g(x)}{f(x)+g(x)}$. If $f(1) = 1$, $g(1) = 2$, $f'(1) = 3$ and $g'(1) = 4$, then find $h'(1)$. Answer: _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the derivatives of the following functions.

a. $f(x) = \frac{\sqrt{2x+1}}{x^2-1}$

b. $g(x) = (3x+1)^4(x^2-x+1)^3$

2. (10 points) The quantity demanded per month, x , of a certain make of tablet PC is related to the average unit price, p (in dollars), of tablet PCs by the equation

$$x = f(p) = \frac{100}{9} \sqrt{810000 - p^2}$$

It is estimated that t months from now, the average price of a tablet PC will be given by

$$p(t) = \frac{400}{1 + \frac{1}{8}\sqrt{t}} + 200 \quad (0 \leq t \leq 60)$$

dollars. Find the rate at which the quantity demanded per month of the tablet PCs will be changing 16 months from now.

3. (10 points) The demand function for a product is given by

$$d = x^2 - 16x + 60 \quad (0 \leq x \leq 6)$$

and the corresponding supply function is given by

$$s = x^2 + 3x + 3 \quad (0 \leq x \leq 6)$$

where d and s are in **dollars** and s is measured in units of a **thousand**.

a. Sketch the graphs of those functions in **ONE** Cartesian coordinate system (4 points).

b. Find the equilibrium quantity and price (6 points).

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4. (10 points) Find the value of

$$\lim_{h \rightarrow 0} \frac{\sqrt{1+h} - 1}{\sqrt{1-h} - 1}.$$

You may lose points if you explain things badly.

5. (10 points) Suppose that a manufacturer of laptops has a monthly fixed cost 20,000 dollars and a variable cost $500x + 200\sqrt{x}$ dollars where x denotes the quantity.
- Find the total monthly cost function $C(x)$ (2 points).
 - Find average cost function $\bar{C}(x)$ and marginal average cost function (4 points).
 - Evaluate $\lim_{x \rightarrow \infty} \bar{C}(x)$ and interpret your result (4 points).
6. (10 points) The concentration of a certain drug in a patient's bloodstream t hours after injection is given by

$$C(t) = \frac{t}{t^2 + 2}$$

- Find the rate at which the concentration of the drug is changing with respect to time (2 points).
- How fast is the concentration changing 1 hour and 3 hours after the injection (4 points)?
- Evaluate $\lim_{t \rightarrow \infty} C(t)$, interpret your result (4 points).

(試題結束)