

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。

(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. A function $y = f(x)$ is integrable on the closed interval $[-1, 1]$, then $y = f(x)$ is continuous on $[-1, 1]$.
2. Let $y = g(x)$ be continuous on the closed interval $[3, 5]$. Then the range of g is a set of a single real number or a closed interval.
3. Let $y = f(x)$ be integrable on the closed interval $[a, b]$. Then there exists a number c in $[a, b]$ such that $\int_a^b f(x) dx = f(c)(b - a)$.
4. A function that is neither increasing nor decreasing can not be one-to-one and hence does not have an inverse.
5. If $y = f(x)$, f is increasing and differentiable, and $\Delta x > 0$, then $\Delta y \geq dy$.
6. $\int_a^b f(x)g(x) dx = \left[\int_a^b f(x) dx \right] \left[\int_a^b g(x) dx \right]$.
7. The integral $\int_0^2 (2x - 1)dx$ is the area of the region bounded by the graph of $f(x) = 2x - 1$ and the x -axis for $0 \leq x \leq 2$.
8. $|\cos x - \cos y| \leq |x - y|$ for all x and y .
9. If $(c, f(c))$ is a point of inflection of the graph of f , then either $f''(c) = 0$ or f'' does not exist at $x = c$.

(下頁還有試題)

10. Let $f(x)$ be a continuous function on an open interval I containing a and $F(x) = \int_a^x f(t) dt$ for $x \in I$. Then $F(x)$ is differentiable on I .

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Find the average value of $f(x) = \tan x$ on the interval $\left[0, \frac{\pi}{4}\right]$.

Answer : _____.

2. Let

$$F(x) = \int_{\pi}^{\ln x} \cos(e^t) dt.$$

Find $F'(\pi)$. Answer : _____.

3. Find $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{3i}{n}\right) \left(\frac{3}{n}\right)$.

Answer : _____.

4. Find $\int \frac{3^{2x}}{1 + 3^{2x}} dx$.

Answer : _____.

5. Find the limit

$$\lim_{x \rightarrow \infty} \left(4x - \sqrt{16x^2 + x}\right).$$

Answer : _____.

6. Find the definite integral

$$\int_1^4 (3 - |x - 3|) dx.$$

Answer : _____.

7. Find an equation of the tangent line to the graph of $y = 3x \arcsin x$ at the point

$\left(\frac{1}{2}, \frac{\pi}{4}\right)$. Answer : _____.

8. Find the absolute maximum for $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Find the derivative of the function $f(x) = x^{e^x}$ on the interval $(0, \infty)$.
2. (10 points) Find the area of the region bounded by the graph of $y = e^x \cos(e^x)$, the x -axis, and the vertical lines $x = -1$ and $x = 0$.
3. (10 points) Let f be continuous on the closed interval $[a, b]$, and differentiable on (a, b) . If $f'(x) > 0$ on (a, b) , then f is increasing on $[a, b]$.
4. (10 points) Let $f(x) = 3x^{2/3} - 2x$. Find **a.** its domain, **b.** critical numbers, **c.** intervals of increasing/decreasing, **d.** relative maximum/minimum values, **e.** intervals of concavity, **f.** inflection points, **g.** asymptotes. **h.** Then sketch the graph of $f(x)$.

5. (10 points)

a. Show that

$$f(x) = \int_0^{1/x} \frac{1}{t^2 + 1} dt + \int_0^x \frac{1}{t^2 + 1} dt$$

is constant for $x > 0$.

b. Show that

$$f(x) = \int_2^x \sqrt{1 + t^2} dt$$

is one-to-one and find $(f^{-1})'(0)$.

6. (10 points) Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?

(試題結束)