考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。 (不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If $h(x) = f(x^2)$, then $h''(x) = (2x)^2 f''(x^2)$.

2. If $A = f(\sqrt{x})$, then the percentage change in A is approximately equal to $\dfrac{50 f'(\sqrt{x})}{\sqrt{x} f(\sqrt{x})}$.

3. The relative minimum of f is always less than or equal to the relative maximum of f.

4. If $x = c$ is a critical number of f and $f'(x)$ changes sign from negative to positive as we move across $x = c$, then f has a relative minimum at $x = c$.

5. If $x < y$, then $\left(\dfrac{1}{e}\right)^x > \left(\dfrac{1}{e}\right)^y$.

6. Suppose $P(t)$ represents the population of bacteria at time t, and suppose $P'(t) > 0$ and $P''(t) < 0$; then the population is increasing at an increasing rate.

7. If c is a critical number of f where $a < c < b$ and $f''(x) < 0$ on (a, b), then f has relative maximum at $x = c$.

8. If $(c, f(c))$ is an infection point of f and if f' is defined and continuous at $x = c$, then $f'(c)$ is a relative extremum of f'.

(下頁還有試題)

9. Let $f(x) = \begin{cases} 1+x & , x \in [-1,0) \\ 0 & , x = 0 \\ x-1 & , x \in (0,1] \end{cases}$, then f has absolute extrema on $[-1,1]$.

10. $\ln(x^2 e^3) = 2\ln|x| + 3$.

填充題 **Short answer questions** (40 points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. The volume of a right-circular cylinder of radius r and height h is $V = \pi r^2 h$. Suppose the radius and height of the cylinder are changing with respect to time t. At a certain instant of time, the radius and height of the cylinder are 2 and 4 in. and are increasing at the rate of 0.1 and 0.3 in./sec, respectively. How fast is the volume of the cylinder increasing? Answer:_____.

2. Find the absolute extrema, if any, of $f(x) = x - 3x^{1/3}$ on the interval $[-3, 27)$. Answer:_____.

3. An apple orchard has an average yield of 36 bushels of apples/tree if the tree density is 24 trees/acre. For each unit increase in tree density, the yield decreases by 2 bushels/tree. How many trees should be planted in order to maximize the yield? Answer:_____.

4. Consider the equation $\sqrt{xy} = \frac{1}{4}x + y^2$. Use implicit differentiation to find the derivative dy/dx at $x = 4$, $y = 1$. Answer:_____.

5. Find the differential of the function $f(x) = (4x^2 - x)^{1/3}$.
   Answer:_____.

6. Find all horizontal asymptotes of the graph of the function $f(x) = \dfrac{|x|x}{x^2 + 1}$.
   Answer:_____.

7. Let $f(x) = x^{1/3}$. Find the inflection point(s), if any, of f.
   Answer:_____.

8. Find the value of k so that $2^x = e^{kx}$ for all x. Answer:_____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) The base of a 13-ft ladder leaning against a wall begins to slide away from the wall. At the instant of time when the top is 12 ft from the ground, the base is moving at the rate of 8 ft/sec. How fast is the top of the ladder sliding down the wall at that instant of time?

2. (*10* points) Let $f(x) = \dfrac{x-1}{x^2}$. Find the relative extrema, the inflection points, and the vertical and horizontal asymptotes, if any, of f. Then sketch the graph of f.

3. (*10* points) The quantity demanded each month of the Sicard sports watch is related to the unit price by the equation $p = \dfrac{50}{0.01x^2 + 1}$, $0 \leq x \leq 20$, where p is measured in dollars and x is measured in units of a thousand. To yield a maximum revenue, how many watches must be sold?

4. (*10* points) Find $\dfrac{d^2y}{dx^2}$ if $xy - y^3 = 4$ in terms of x and y.

5. (*10* points) Use a differential to approximate the quantity $(80.9)^{1/4}$.

6. (*10* points) Betty Moore company requires that its corned beaf hash containers have a capacity of 128 cubic inces, have the shape of a a right circular cylinder, and made of aluminum. Determine the radius and height of the container that requires the least amount of metal.

(試題結束)