

考試時間 120 分鐘，題目卷為兩張紙，共三頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。 **Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (20 points)，請答 **T** (True) 或 **F** (False)。每題 2 分。
(不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. Suppose $f(x)$ is a function on the closed interval $[1, 3]$ and $f(1) \cdot f(3) < 0$, then there exists a number c in $(1, 3)$ such that $f(c) = 0$.
2. A function $y = f(x)$ is integrable on the closed interval $[-1, 1]$, then $y = f(x)$ is continuous on $[-1, 1]$.
3. The integral $\int_0^2 (2x - 1) dx$ is the area of the region bounded by the graph of $f(x) = 2x - 1$ and the x -axis for $0 \leq x \leq 2$.
4. $\lim_{x \rightarrow 0^+} \frac{1 - \cos x}{x + x^3} = \lim_{x \rightarrow 0^+} \frac{\sin x}{1 + 3x^2} = \lim_{x \rightarrow 0^+} \frac{\cos x}{6x} = \infty$.
5. Since $\lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2 + 1} dx = 0$, we have

$$\int_{-\infty}^{\infty} \frac{2x}{x^2 + 1} dx = \lim_{b \rightarrow \infty} \int_{-b}^b \frac{2x}{x^2 + 1} dx = 0.$$

6. $\frac{8x^3 + 13x}{(x^2 + 2)^2}$ can be put in the form $\frac{Ax + B}{x^2 + 2} + \frac{Cx + D}{(x^2 + 2)^2}$.
7. If $f'(x)$ is continuous on $[0, 1]$ and $f(0) = 0$, then

$$\int_0^\alpha \sqrt{1 + [f'(x)]^2} dx \geq \sqrt{\alpha^2 + [f(\alpha)]^2}$$

for $0 \leq \alpha \leq 1$.

(下頁還有試題)

8. $\int_1^{\infty} \frac{1}{x\sqrt{\frac{1}{2}}} dx$ converges.
9. If $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 1$, then $\lim_{x \rightarrow \infty} [f(x) - g(x)] = 0$.
10. $\int_{-1}^1 \frac{1}{x^2} dx = -2$.

填充題 **Short answer questions** (40 points), 每題 5 分。

(不需詳列過程, 僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $f(x) = \int_2^x \sqrt{1+t^2} dt$. Find $(f^{-1})'(0)$.
Answer : _____.
2. Find $\int_0^{\pi/2} \cos x \cos 2x dx$.
Answer : _____.
3. Evaluate $\lim_{x \rightarrow 0^+} (\sin x)^x$.
Answer : _____.
4. Find the volumes of the solid generated by revolving the region bounded by the graphs of the equations $y = \sqrt{x}$, $y = 0$, $x = 4$ about the line $x = 4$.
Answer : _____.
5. Find $\int_{-1}^{1/2} \frac{1}{\sqrt{8-2x-x^2}} dx$.
Answer : _____.
6. Find the area of the region bounded by the graphs of $x = 3 - y^2$ and $x = y + 1$.
Answer : _____.
7. Evaluate $\int_{\sqrt{3}}^2 \frac{\sqrt{x^2-3}}{x} dx$.
Answer : _____.
8. Evaluate $\int_1^{\infty} (1-x)e^{-x} dx$.
Answer : _____.

(下頁還有試題)

計算問答證明題 **Please show all your work** (60 points), 每題 10 分, 請依題號順序依序寫在答案卷上, 可以用中文或英文作答。請詳列計算過程, 否則不予計分。需標明題號但不必抄題。

1. (10 points) Given $xy - 1 = 2x + y^2$, find the value of $\frac{d^2y}{dx^2}$ at the point $\left(\frac{-1}{2}, 0\right)$.

2. (10 points) Evaluate $\int \frac{\sqrt{x}}{x-1} dx$.

3. (10 points) Evaluate $\int_0^{\infty} \frac{dx}{2\sqrt{x}(x+1)}$.

4. (10 points) Find the integral.

a. $\int_0^{\frac{\pi}{4}} \sec^5 x \tan^3 x dx$.

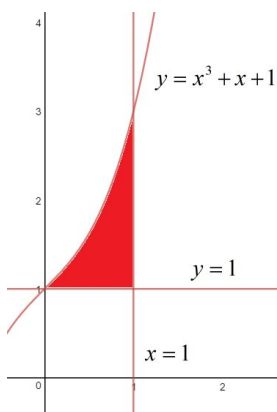
b. $\int_0^{\frac{\pi}{2}} \sin^3 x \cos^4 x dx$.

5. (10 points) Let $f(x) = \frac{x^3}{12} + \frac{1}{x}$, $1 \leq x \leq 2$.

a. Find the length of the graph of f .

b. Find the area of the surface generated by revolving the curve $y = f(x)$ about the x -axis.

6. (10 points) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$.



(試題結束)