管院微積分聯合教學　會考三　**Jan 10, 2017**

考試時間 120 分鐘，題目卷為兩張紙，共四頁，滿分 120 分。所有題目的答案都請依題號順序依序寫在答案卷上，而是非與填充題必須寫在第一頁。答案卷務必寫學號、姓名，題目卷不必繳回。考試開始 30 分鐘後不得入場，開始 40 分鐘內不得離場。考試期間禁止使用字典、計算機、任何通訊器材並請勿自行攜帶任何紙張，違者成績以零分計算，監試人員不得回答任何關於試題的疑問。**Questions are to be answered on the answer sheet provided.**

是非題 **True or False** (*20* points)，請答 **T** (True) 或 **F** (False)。每題 2 分。 (不需詳列過程，請依題號順序依序寫在答案卷第一頁上。)

1. If f is a continuous function on an open interval (a, b) and if $f(a)$ and $f(b)$ have opposite signs, then there is at least one solution of the equation $f(x) = 0$ in the interval (a, b).

2. If $x < y$, then $\left(\frac{1}{e}\right)^x > \left(\frac{1}{e}\right)^y$.

3. The effective interest rate $\hat{r}_{\text{eff}}$ that corresponds to a nominal interest rate r per year compounded continuously is given by $\hat{r}_{\text{eff}} = e^r - 1$.

4. If $x^2 + \ln y = 10$, then $\dfrac{dy}{dx} = -2xy$.

5. $\displaystyle\int \left(\sqrt{x}e^x + \frac{2}{3}x^{3/2}e^x\right)\, dx = \frac{2}{3}x^{3/2}e^x + C.$

6. $\displaystyle\int \frac{1}{x^2+1}\, dx = \ln|x^2+1| + C.$

7. If f is continuous on $[a, b]$ and $a < c < b$, then $\displaystyle\int_c^b f(x)\, dx = \int_c^a f(x)\, dx + \int_a^b f(x)\, dx$.

8. The area of the region bounded by the graphs of the functions $f(x) = x^2$ and $g(x) = x^{1/3}$ is given by $\displaystyle\int_0^1 \left(x^{1/3} - x^2\right)\, dx$.

(下頁還有試題)

9. If f and g are continuous on $[a, b]$ and $f(x) \geq g(x)$ for all x in $[a, b]$, then $\int_a^b [f(x) - g(x)]\, dx \geq 0$.

10. The consumers' surplus is given by $CS = \int_0^{\bar{x}} D(x)\, dx - \bar{p}\bar{x}$, where $D(x)$ is the demand function, $\bar{p}$ is the unit market price, and $\bar{x}$ is the quantity sold.

填充題 **Short answer questions** (*40* points)，每題 5 分。

(不需詳列過程，僅將答案依題號順序依序寫在答案卷第一頁上即可。)

1. Let $h(x) = \dfrac{f(x)g(x)}{f(x) + g(x)}$. If $f(1) = 3$, $g(1) = 1$, $f'(1) = 4$ and $g'(1) = 2$, then find $h'(1)$. Answer:_____.

2. Find the derivative of the function $f(x) = \dfrac{2 \ln x}{x}$. Answer:_____.

3. Evaluate $\int_0^1 \dfrac{e^x}{1 + e^x}\, dx$. Answer:_____.

4. How long will it take \$5,000 to grow to \$15,000 if the investment earns interest at the rate of 4%/year compounded semiannually. Answer:_____.

5. Find the absoluate minimum of the function $f(x) = (1 - 2x)e^{-x}$ on $[0, \infty)$. Answer: _____.

6. The daily marginal profit function associated with producing and selling TexaPep hot sauce is $P'(x) = -0.000006x^2 + 6$, where x denotes the number of cases (each case contains 24 bottles) produced and sold daily, and $P'(x)$ is measured in dollars per unit. The fixed cost is \$500. What is the total profit realizable from producing and selling 1000 cases of TexaPep per day? Answer:_____.

7. Evaluate the definite integral $\int_{-1}^1 \dfrac{x}{1 + x^{2/3}}\, dx$. Answer:_____.

8. Aiso deposits \$150/month in a savings account paying 5%/year compounded continuously. Estimate the amount that will be in his account after 15 years. Answer:_____.

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計算問答證明題 **Please show all your work** (*60* points)，每題 10 分，請依題號順序依序寫在答案卷上，可以用中文或英文作答。請詳列計算過程，否則不予計分。需標明題號但不必抄題。

1. (*10* points) The base of a 13-ft ladder leaning against a wall begins to slide away from the wall. At the instant of time when the top is 12 ft from the ground, the base is moving at the rate of 8 ft/sec. How fast is the top of the ladder sliding down the wall at that instant of time?

2. (*10* points) The length (in centimeters) of a typical Pacific halibut t years old is approximately

$$f(t) = 200(1 - 0.956e^{-0.18t})$$

a. What is the length of a typical 5-year-old Pacific halibut?

b. How fast is the length of a typical 5-year-old Pacific halibut increasing?

c. What is the maximum length a typical Pacific halibut can attain?

3. (*10* points) The rate of change of the unit price p (in dollars) of Apex women's boots is given by

$$p'(x) = \frac{-250x}{(16 + x^2)^{3/2}}$$

where x is the quantity demanded daily in units of a hundred. Find the demand function for these boots if the quantity demanded daily is 300 pair ($x = 3$) when the unit price is \$55/pair.

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4. (*10* points) The production of oil (in millions of barrels per day) extracted from oil sands in Canada is projected to grow according to the function

$$P(t) = \frac{4.76}{1 + 4.11e^{-0.22t}}, \quad 0 \leq t \leq 20,$$

where t is measured in years, with $t = 0$ corresponding to 2005. What is the expected total production of oil from oil sands over the years from 2005 until 2025 (t=20)?

5. (*10* points) The concentraction of a certain drug on a patient's bloodstream t hr after injection is $C(t) = \dfrac{0.2t}{t^2 + 1}$ mg/cm^3. Determine the average concentration of the drug in the patient's bloodstream over the first 5 hr after the drug is injected.

6. (*10* points) In a study conducted by a certain country's Economic Development Board, it was found that the Lorenz curve for the distribution of income of stock-brokers was described by the function $f(x) = \dfrac{11}{12}x^2 + \dfrac{1}{12}x$ and that of high school teachers by the function $g(x) = \dfrac{5}{6}x^2 + \dfrac{1}{6}x$.

a. Compute the coefficient of inequality for each Lorenz curve.

b. Which profession has a more equitable income distribution?

(試題結束)